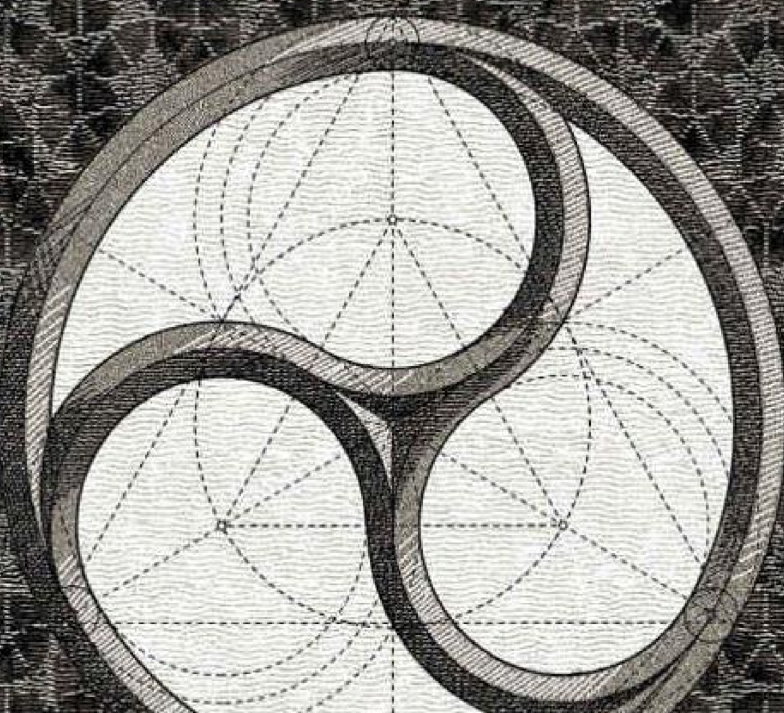
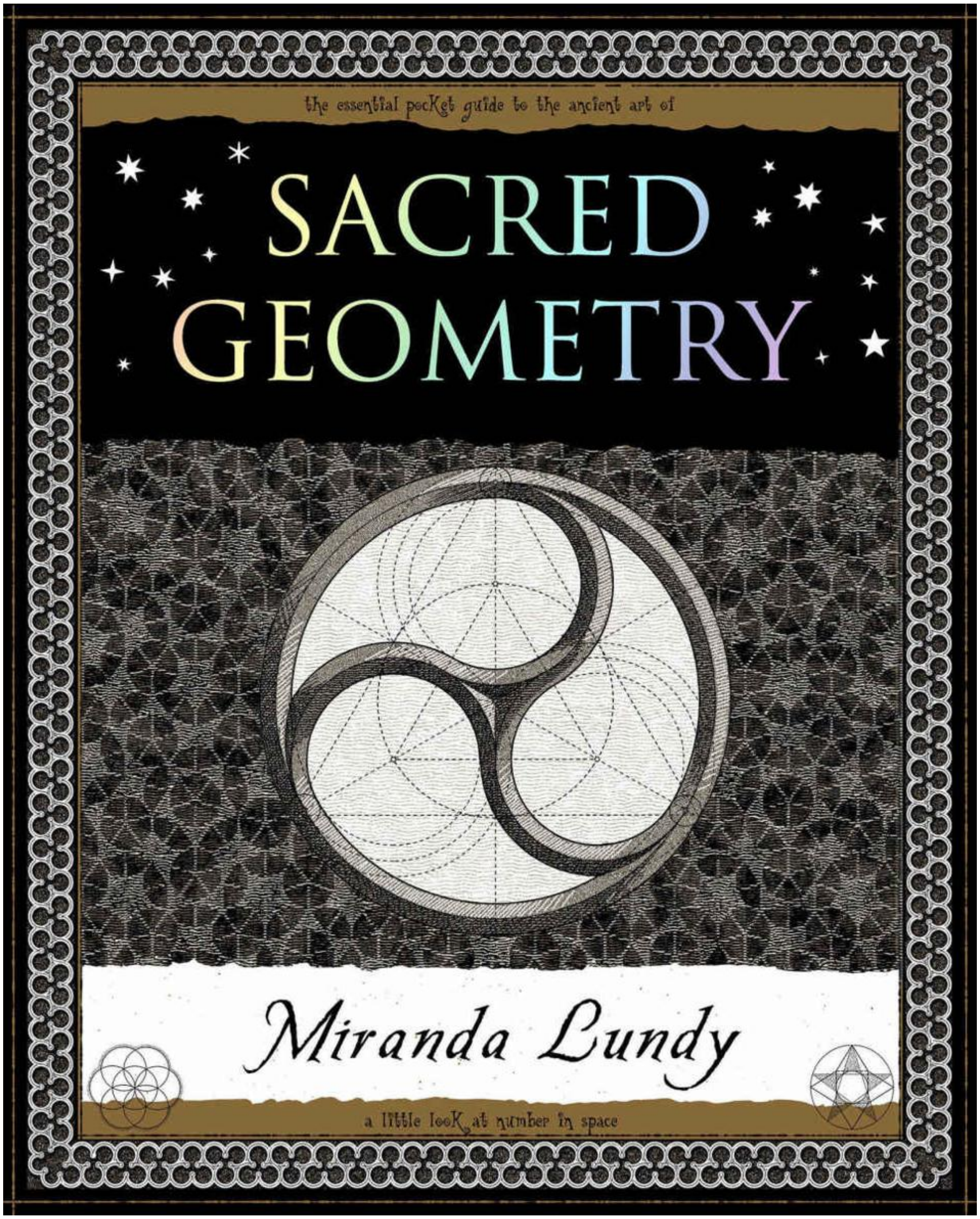
the essential pocket guide to the ancient art of

SACRED GEOMETRY



Miranda Lundy

a little look at number in space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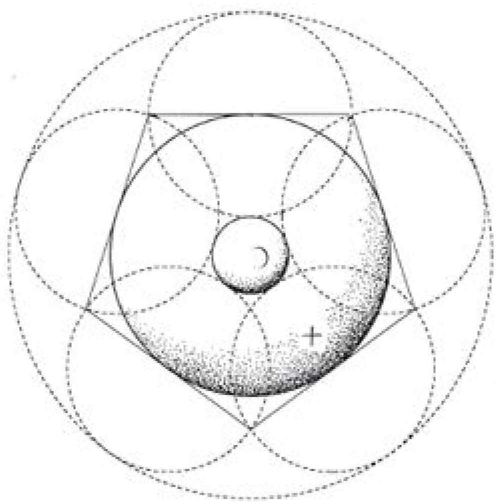

the relative sizes of the Moon and

the Earth defined from a pentagon

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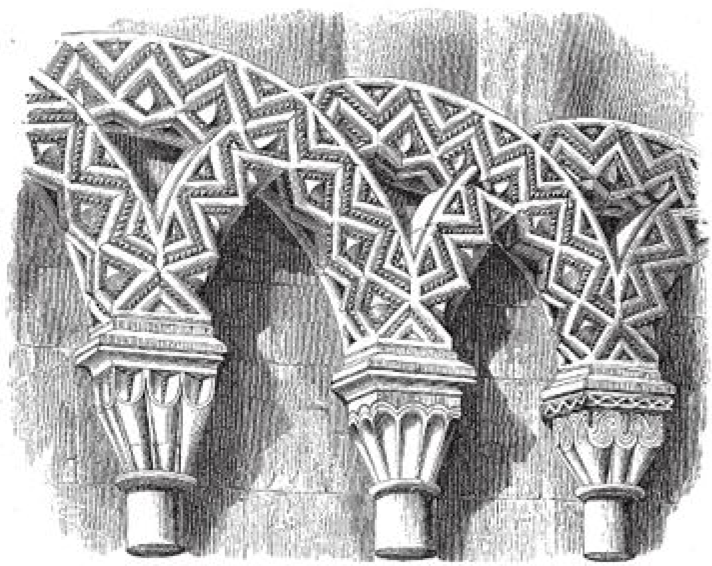
SACRED
GEOMETRY



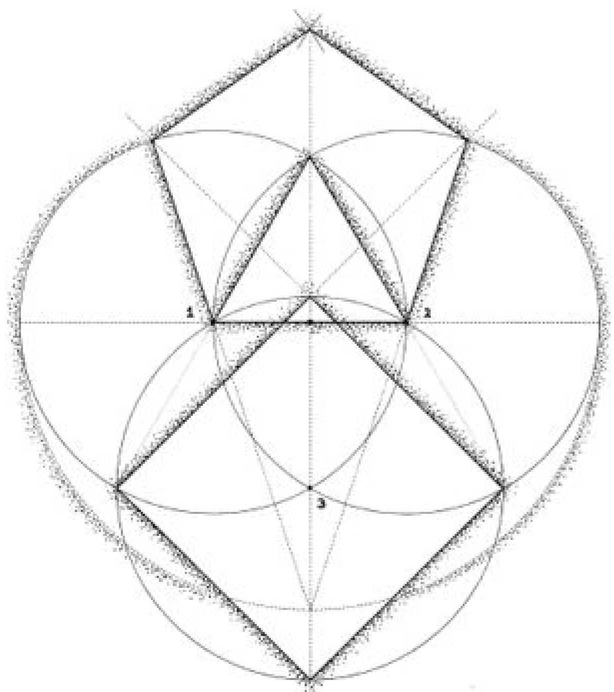
written & illustrated by

Miranda Lundy

To the designers of the future, young and old.

I recommend two other books in this series: "Platonic and Archimedean Solids" by Daud Sutton, which deals with the all-important third dimension, and "Ruler and Compass" by Andrew Sutton, which covers ruler and compass constructions.

My sincere thanks to John Martineau, and to my teachers Prof. Keith Critchlow, John Michell, Dr. Khaled Azzam, Paul Marchant, Robin Heath, Michael Glickman, Dr. Stephan René, Tony Ashton and Buckminster Fuller.



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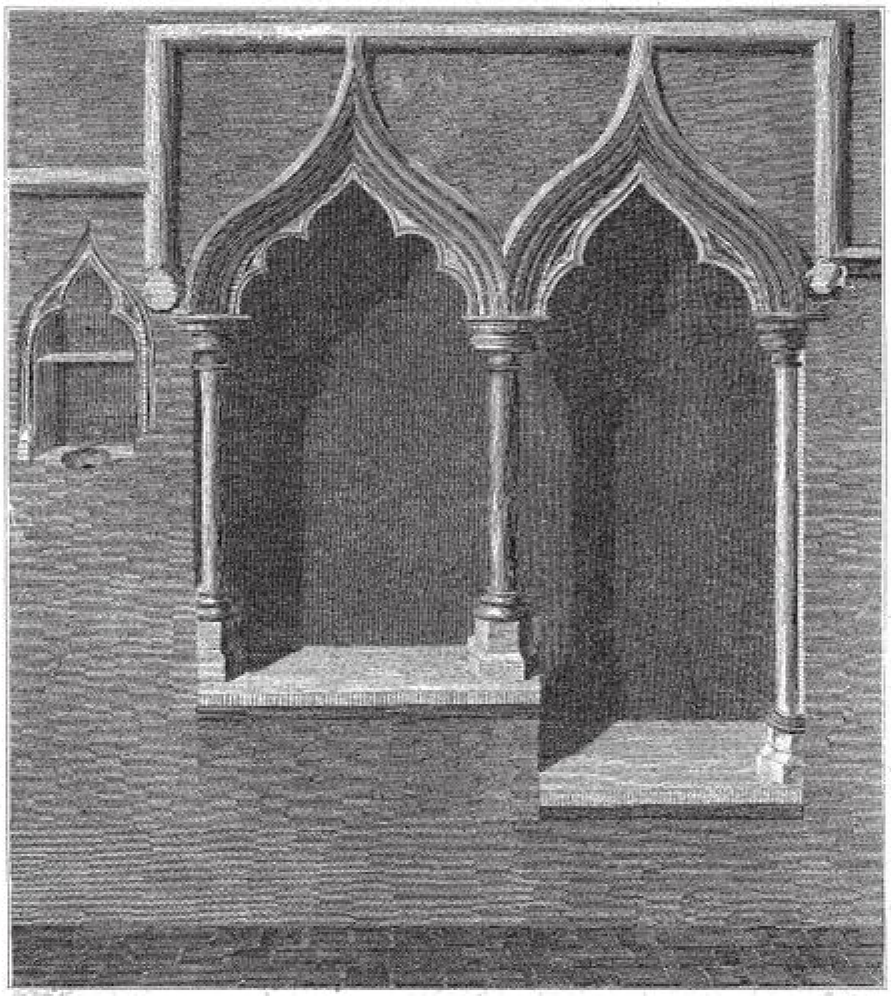


Seats in Norwoods chantry, in Milton church, Kent.

Introduction

Sacred Geometry charts the unfolding of number in space and differs from mundane geometry in the sense that its moves, concepts and products are regarded as having symbolic value and meaning. Thus, like a good melody, the study and practice of geometry can facilitate the evolution of the soul. As we shall see, the basic journey is from the single point, into the line, out to the plane, through to the third dimension and beyond, eventually returning to the point again, watching what happens on the way.

These pages cover the elements of two-dimensional geometry—the unfolding of number on a flat surface. The three-dimensional geometrical story is told in another book in this series. The material is this book has been used for a very long time indeed as one introduction to metaphysics. Like the elements of its sister subject, music, it is an aspect of revelation, a bright indisputable shadow of Reality and a creation myth in itself.

Number, Music, Geometry and the study of patterns in the Heavens are the four great Liberal Arts of the ancient world dealing with quanta, or whole numbers. These simple universal languages are as relevant today as they have always been, and may be found in all known sciences and cultures without disagreement. Just above the entrance to Plato's Academy was a sign: "Let none ignorant of geometry enter here." So let's do some research.

Point, Line and Plane

none, one and two dimensions

Begin with a sheet of paper. The point is the first thing that can be done. It is without dimension and is not in space. Without an inside or an outside, the point is the source for all which now follows. The point is represented (*below*) as a small circular dot.

The first dimension, the line, comes into being as the One emerges into two principles, active and passive (*below right*). The point chooses somewhere 'outside' of itself, a direction. Separation has occurred and the line comes into being. A line has no thickness, and it is sometimes said that a line has no end.

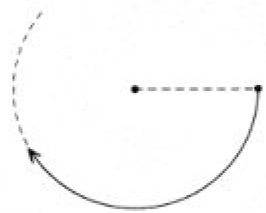
Three 'ways' now become apparent (*opposite*):

i) With one end of the line stationary, or passive, the other is free to rotate and describe a circle, representing Heaven.

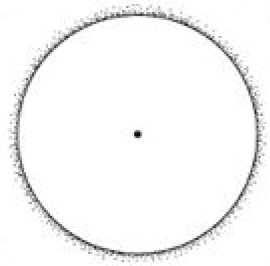
ii) The active point can move to a third position equidistant from the other two, thus describing an equilateral triangle.

iii) The line can produce another which moves away until distances are equal to form a square, representing Earth.

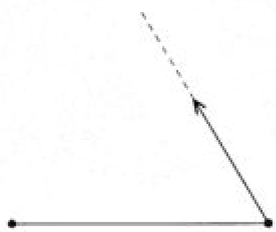
Three forms, circle, triangle and square have manifested. All are rich in meaning. Our journey has begun.



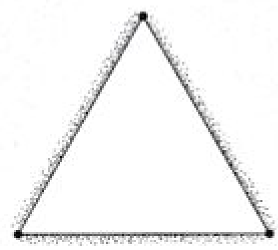
rotate a line



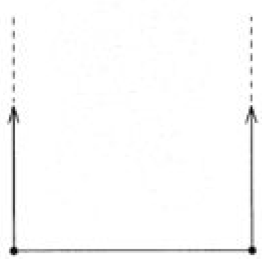
create a circle



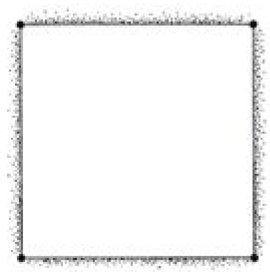
move a vertex



make a triangle



translate a line



produce a square

Sphere, Tetrahedron & Cube

from two to three dimensions

Although this book concerns itself primarily with the plane, the three 'ways' are here taken one step further:

i) The circle spins to become a sphere. Something circular remains essentially circular (*top row opposite*).

ii) The triangle produces a fourth point at an equal distance from the other three to produce a tetrahedron. One equilateral triangle has made three more (*central row opposite*).

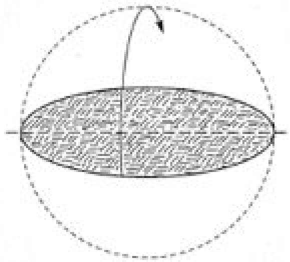
iii) The square lifts a second square away from itself until another four squares are formed and a cube is created (lower row opposite).

Notice how the essential division into circularity, triangularity and squareness from the previous page is preserved.

The sphere is a symbol of the cosmos. Very large and very small things tend to be spherical. Einstein discovered that a point in four dimensions (i.e. you here and now) is a sphere expanding at the speed of light. All we can see of the entire universe is inside an spherical event-horizon. The cube represents the Earth.

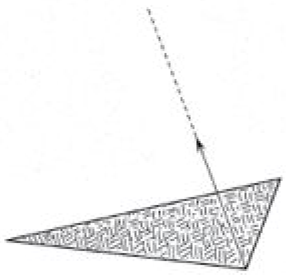
The sphere possesses the smallest surface area for its volume of any possible three-dimensional solid whereas, amongst regular solids, the tetrahedron is the opposite.

A tetrahedron is in fact hiding in a cube—if you draw a single diagonal line on every face of a cube so that they join at the corners, you will have defined the edges of a tetrahedron. Try it!

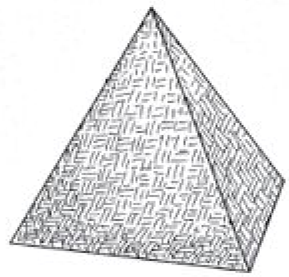


rotate a circle

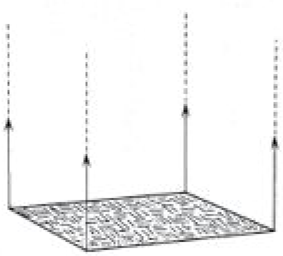
create a sphere



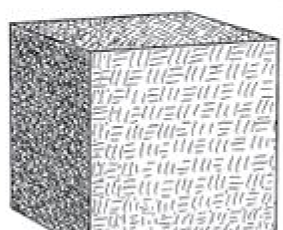
move a vertex



make a tetrahedron



translate a square



produce a cube

One, Two & Three

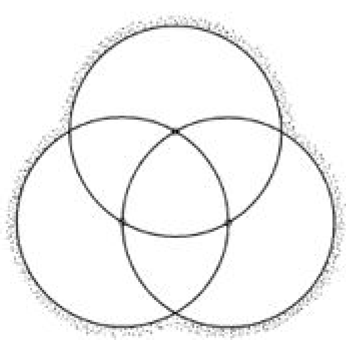
playing with circles

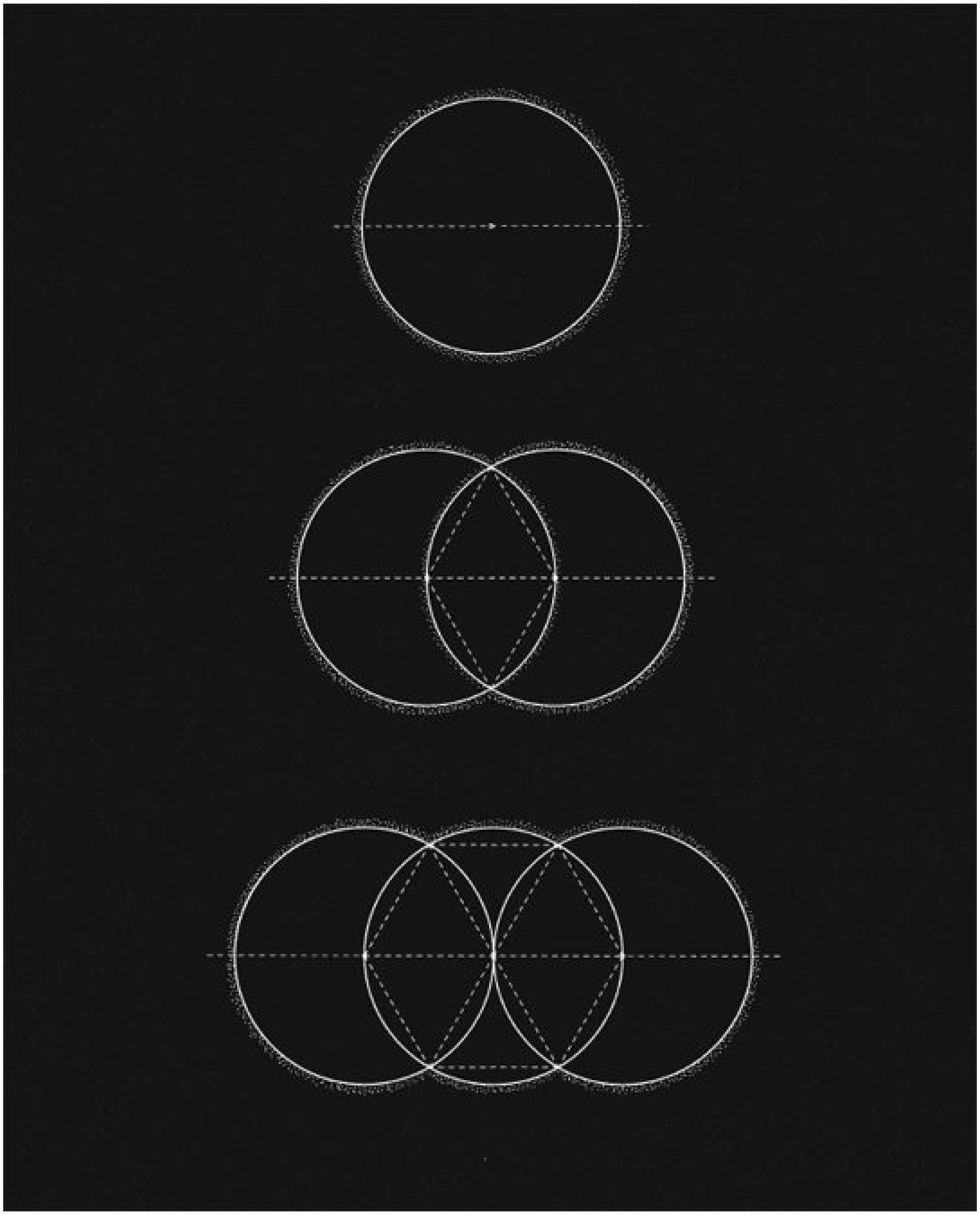
Get a ruler, compasses, something to draw with and something to draw on. Draw a horizontal line across the page. Open the compasses and place the point on the line. Draw a circle (*top*).

Where the circle has cut the line, place the point and draw another circle, the same size as the first. When one circle is drawn over another like this so that their rims pass through each others' centres, then an almond shape, the *vesica piscis,* literally 'fish's bladder', is formed. It is one of the first things that circles can do. Christ is often depicted inside a vesica. Two equilateral triangles have been defined (*opposite centre*).

If a third circle is added on the other side of the forming circle then all six points of a perfect hexagon are defined. Otherwise another circle can be drawn from the top or bottom of the vesica to produce the triangular figure below.

Circles thus effortlessly produce perfect triangles and hexagons.





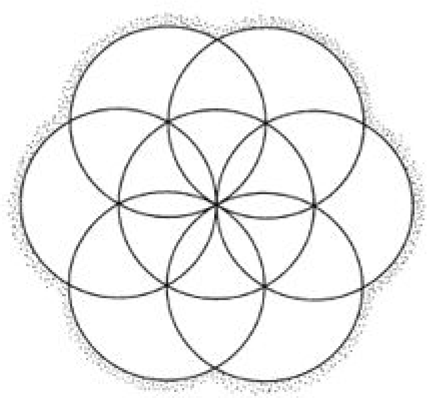
Six Around One

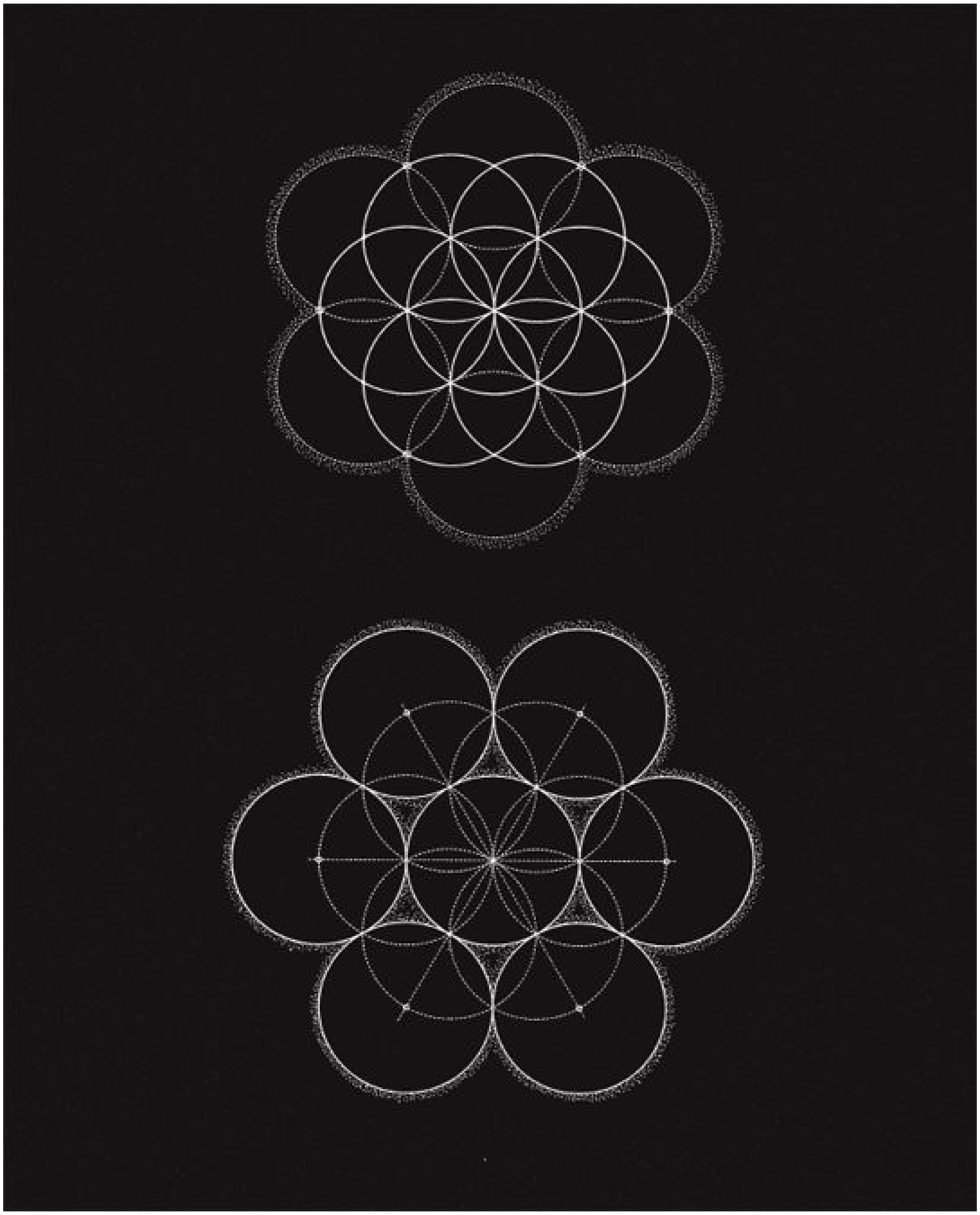
or twelve or even eighteen

The six points of the hexagon give rise to the flower-like pattern shown below. Alternatively it can be drawn by 'walking' a circle around itself—something most children have done at school, whether under instruction or just playing with compasses.

We are now seeking the lower diagram opposite, and need the centres of the six outer circles. One way is to lightly draw the six circles shown dashed above, otherwise we can draw straight lines as shown in the lower diagram. Both ways work.

We can now see that six circles fit around one. We can push oranges, wine-glasses or tennis balls together to see it, yet it is extraordinary really. 'Six around one' is a theme which the Old Testament of the Bible opens on, with the six days of work and the seventh day of rest. There is indeed something very sixy about circles.





Twelve Around One

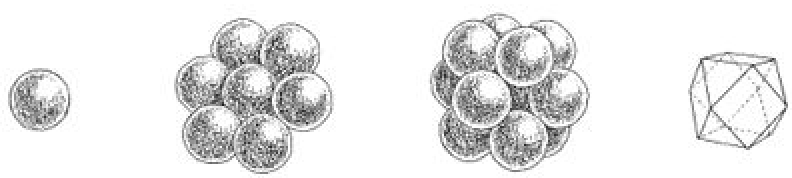
how to draw a dodecagon

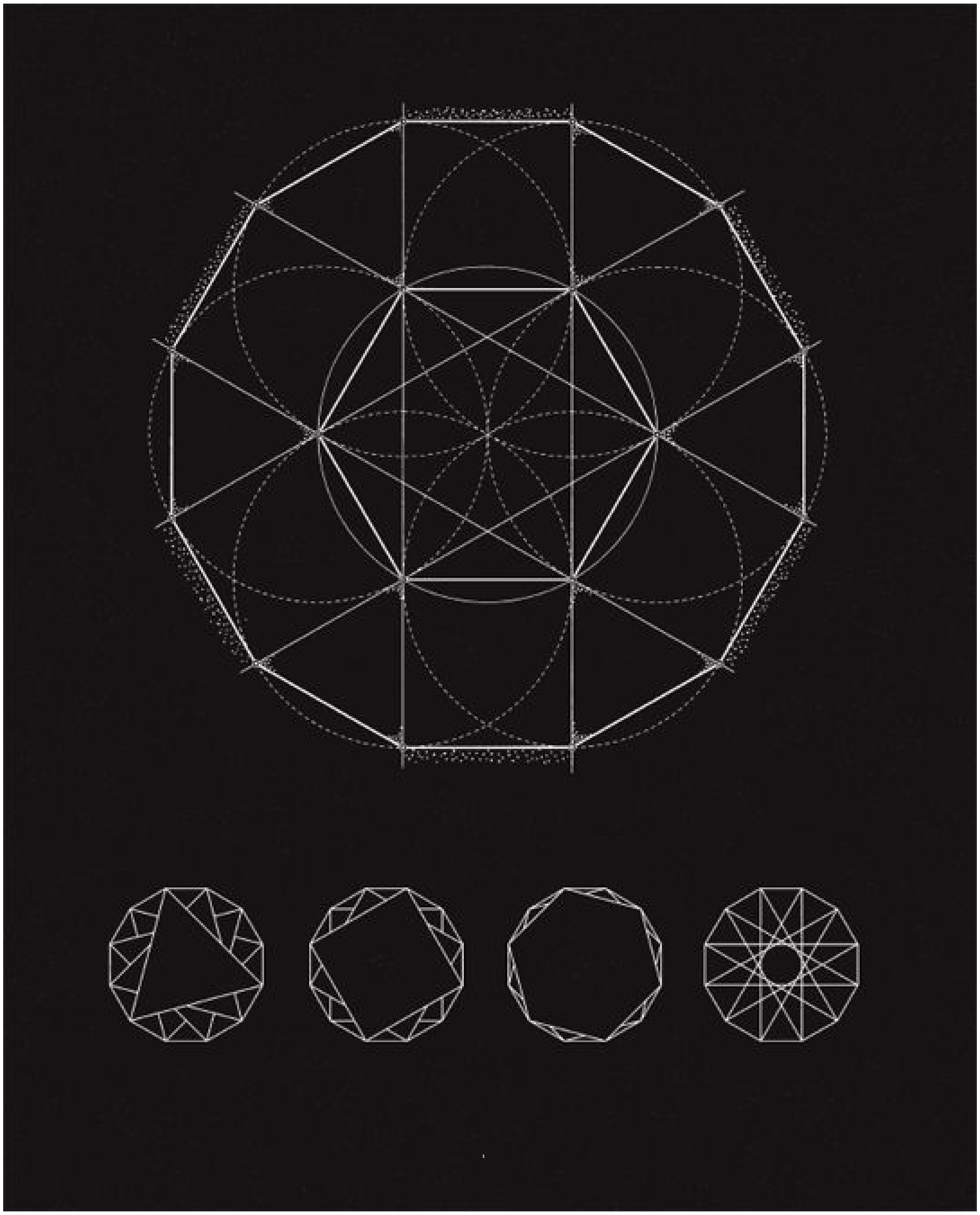
As one produces six, so six produces twelve. The arms of the six-pointed star extend to intersect the outer rims of the six circles to form a perfect overall division of space into twelve parts (*shown opposite*). The perfect twelve-sided polygon is called a dodecagon, which means 'twelve sided'!

A dodecagon is made from six squares and six equilateral triangles fitted around a hexagon - can you see them all opposite? In addition, the twelve divides into three, four and six as four triangles, three squares or two hexagons (*lower row opposite*).

Shown below is the three-dimensional version of the same story. A ball naturally fits twelve others around it so that they all touch the centre and four neighbours. You see this arrangement in apples and oranges in every market stall. The shape is called the cuboctahedron and is closely related to the tetrahedron and cube we saw on page five. Many crystals grow along these lines.

Twelve is the number which fits around one in three dimensions in the same way that six fits around one in two dimensions. The New Testament is a story of a teacher with twelve disciples.





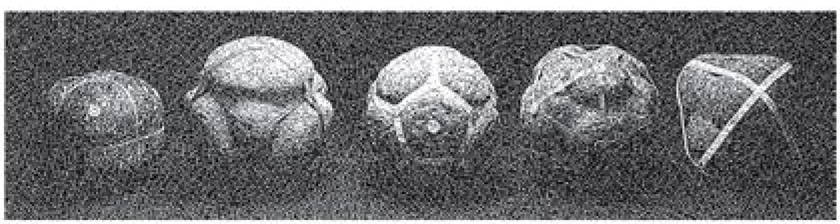
The Five Elements

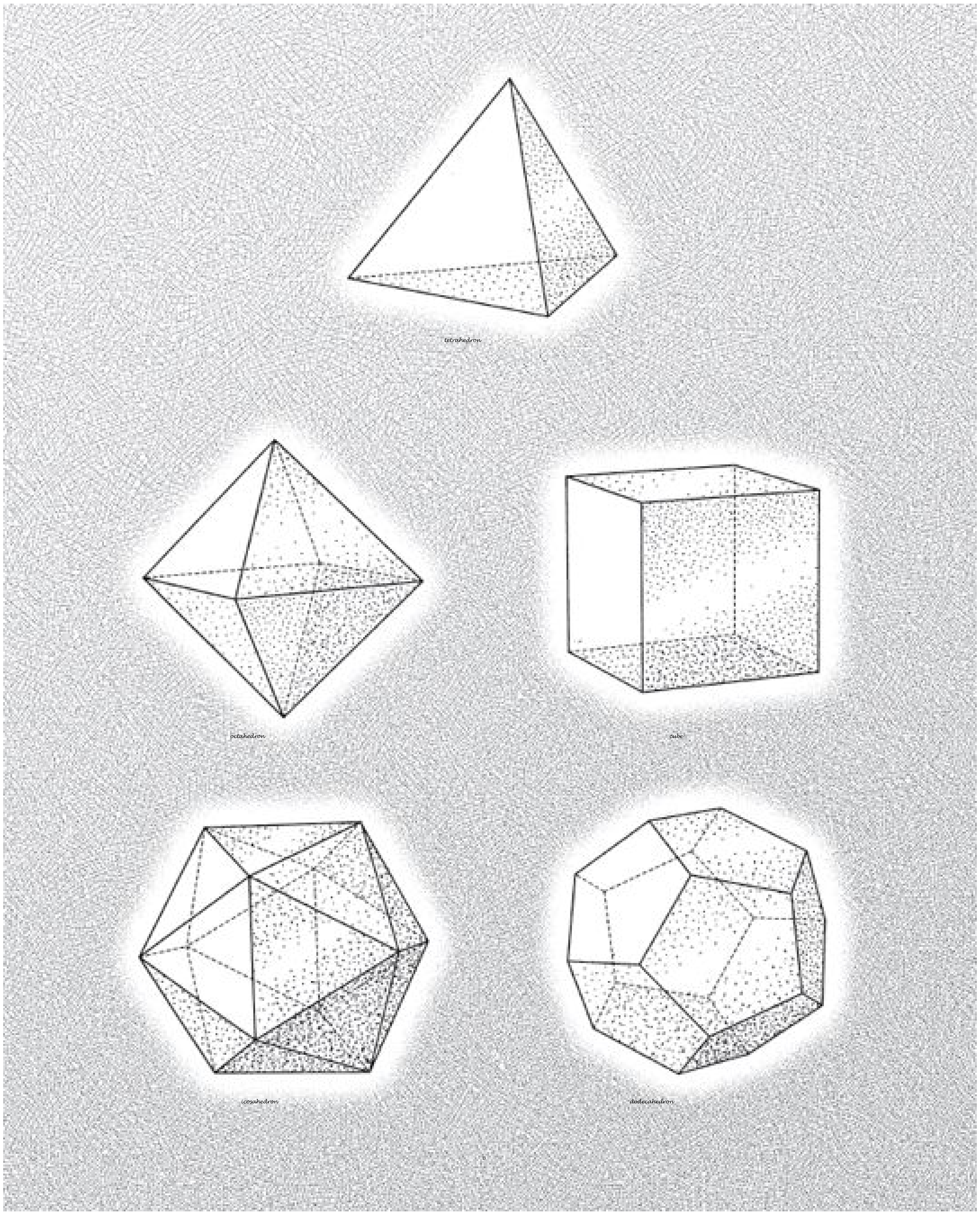
a brief foray into the third dimension

Although outside the scope of this book, it is worth mentioning that there are just five regular three-dimensional solids. Each has equal edges, every face is the same perfect polygon and every point is the same distance from the centre. Known as the five Platonic Solids, they were recognised in the British Isles two thousand years before Plato; 4000-year-old sets of them have been found at stone circles in Aberdeenshire, Scotland (*below, after Critchlow*).

The first solid is the tetrahedron, with four vertices and four faces of equilateral triangles, traditionally representing the element of Fire. The second solid is the octahedron, made from six points and eight equilateral triangles, representing Air. The Cube is the third solid, eight vertices and six square faces, representing Earth. The fourth is the icosahedron, with twelve points and twenty faces of equilateral triangles, the element of Water. The last, and fifth, element is the dodecahedron, which has twenty vertices, representing the mysterious fifth element of Aether.

Notice how beautiful the dodecahedron is, and how it is made of twelve pentagons, perfect five-sided shapes.




tetrahedron
octahedron
cube
icosahedron
dodecahedron

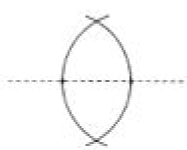
Circling the Square

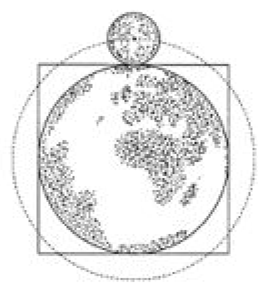
the marriage of heaven and earth

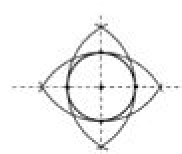
The circle is the shape traditionally assigned to the Heavens, and the square to the Earth. When these two shapes are unified by being made equal in area or perimeter we speak of 'squaring the circle', meaning that Heaven and Earth, or Spirit and Matter, are symbolically combined, or married. Fivefold Man exists between sixfold Heaven and fourfold Earth and Leonardo da Vinci's image opposite also shows how a man's span equals his height, that this measure equals seven of his feet and other important ratios.

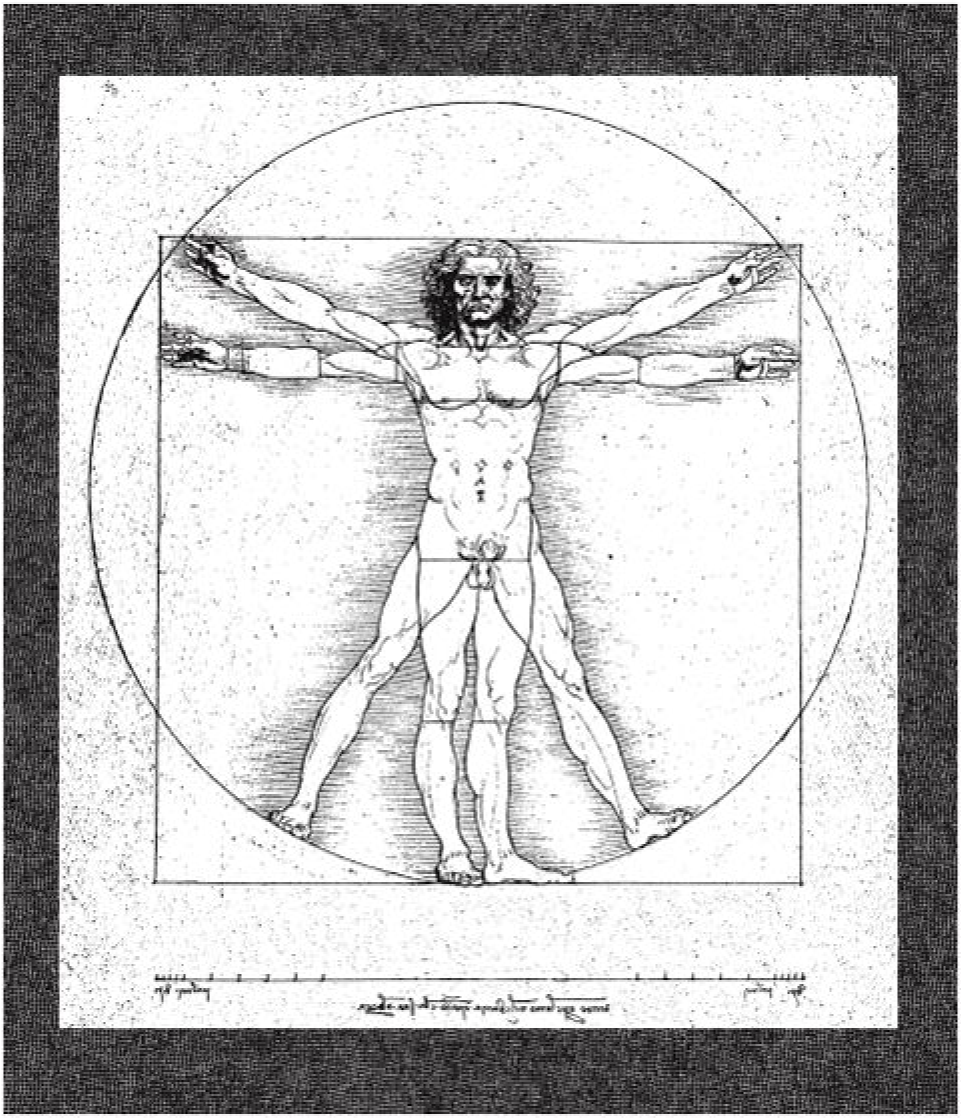
Amazingly, the Earth and the Moon square the circle, for if the Moon (diameter 3) is drawn down to the Earth (diameter 11) then a heavenly circle through the Moon (*dotted below centre*) has radius 7, and so circumference 44, the same as the perimeter of the square around the Earth. This works because π, relating the circumference of a circle to its diameter is practically 22/7. In Leonardo's image the Moon would fit above the man's head.

Also shown (*below left and right*) is a construction for a square using ruler and compasses. Octagons soon follow.







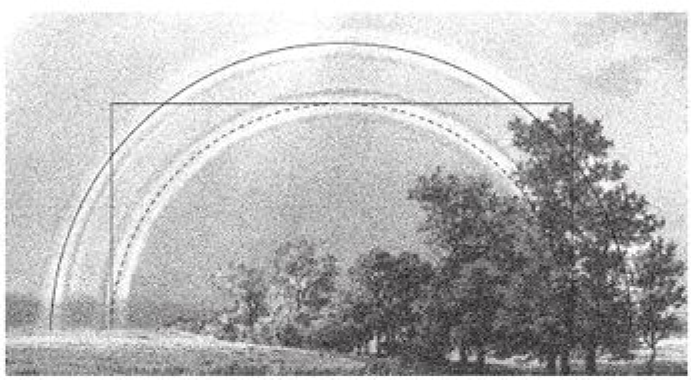


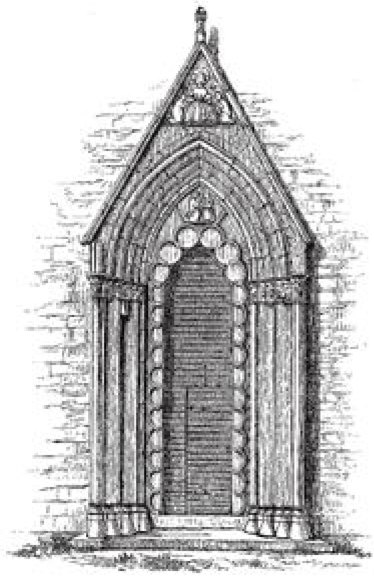
THE CANON

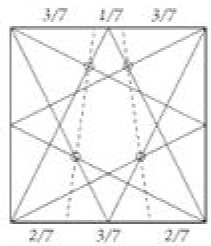
the numbers of the heavens and earth

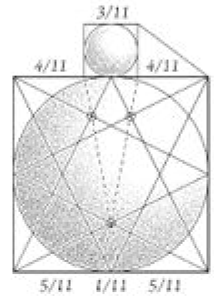
The squaring of the circle as 3:11 is coded in a door at Gerum Church in Gothland, Sweden (*opposite*). Irish and Norse myths abound with tales of 33 warriors (3 x 11). The Sun takes 33 years for a perfect repeat sunrise on the horizon. Jesus dies and is resurrected aged 33. Ramadan occurs every 12 moons and takes 33 years to move around the calendar. Halfway between 3 and 11 is 7 and it is an old secret that the Earth's tilt, often hidden in sacred art as the tilt of a holy head (the Virgin Mary's or the Buddha's), is the diagonal of a rectangle 3 wide and 7 high. Importantly, $^{11}/_{7}$ is the ancient Egyptian value for half of π, so a circle radius *r* has quadrant arc $^{11}/_{7}r$. The Sandreckoner's diagram (*opposite*) is a unique way of dividing a rectangle's edge into canonical fractions (*after Malcolm Stewart*).

A circle and square can also be married by having the same *area*, and a double rainbow, with bows at 41.5° and 52.5°, precisely draws this (*below from Martineau*)—a marriage of Heaven and Earth indeed!

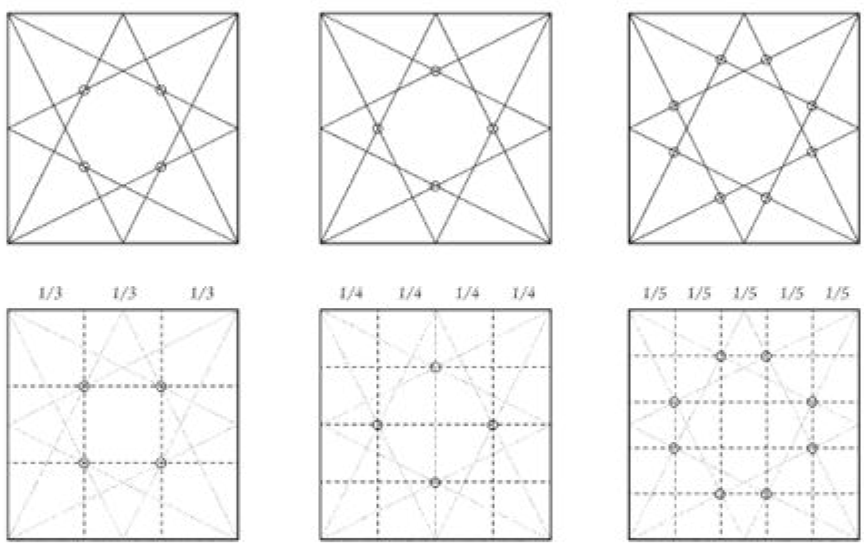








Above and below: The Sandreckoner's diagram. Simply by joining corners to the centers of sides, a square's edge may be exactly divided into 3, 4, 5, and above, 7 and 11 equal parts, which makes it an extremely useful device. The initial lines also produce a plethora of whole number lengths, areas, and shapes, including a multiude of Pythagorean 3-4-5 triangles at various scales.



Pyramid Pie

a marriage of everything

There is perhaps no more famous a geometric object on Earth than the Great Pyramid at Giza in Egypt with its strange passages and enigmatic chambers. The five diagrams opposite show:

1: The 51.8° slope of the Great Pyramid means the square of its height produces an area equal to that of each face.

2: The *golden section* in the pyramid, ϕ = 1.618 (*see page 24*).

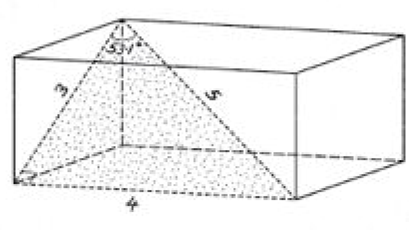
3: *Pi* in the pyramid. Pi, or π, defines the ratio between a circle's circumference and its diameter (3.14159...).

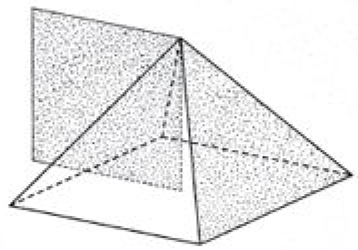
4: The pyramid squaring the circle (*see page 14*).

5: A pentagram defining a 'net' for the pyramid - cut and fold!

The angle of slope, 51.8° is also close to 1/7 of a circle, 51.4°. Geometry means 'Earth measure' and the Pyramid functions as a ridiculously accurate sundial, observatory, surveyor's tool and repository for standard weights and measures. Its perimeter is exactly half a degree of equatorial latitude.

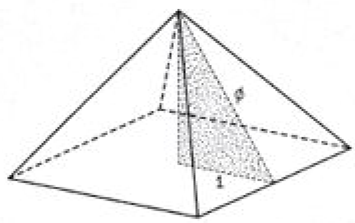
A 3-4-5 triangle fits the shape of the 'King's chamber' (*below*) and gives the angle of slope of the second pyramid at Giza.





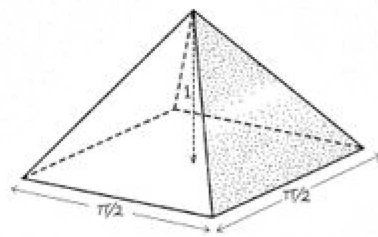
a map of the Earth

square of height = area of face



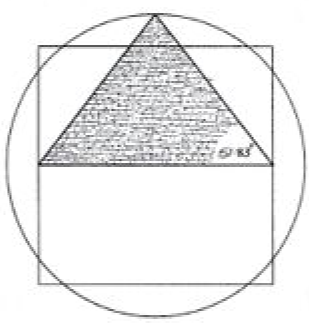
the golden section in the pyramid

cosine of slope = 0.618



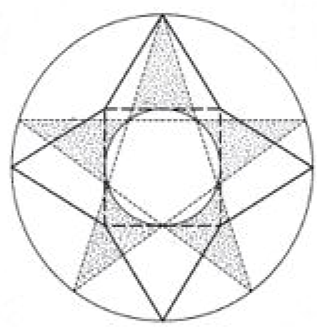
pi in the pyramid

perimeter = 2π height



circumference of circle on height

= perimeter of pyramid base



constructing the pyramid from a

pentagram drawn in a circle

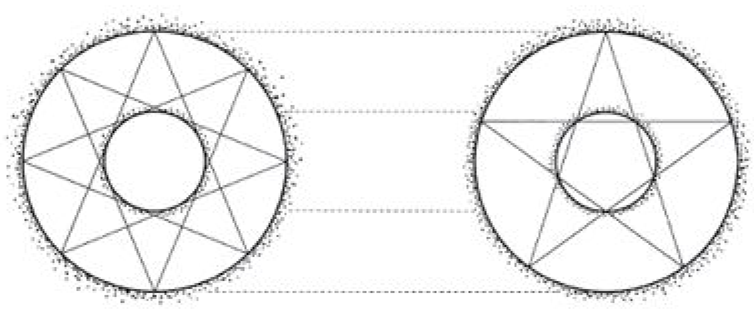
Halflings and Thirdlings

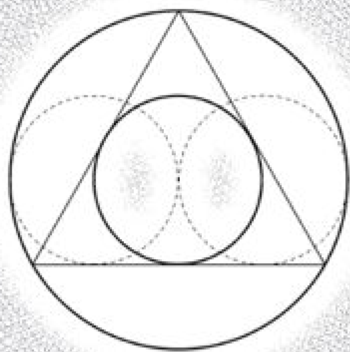
defined by triangles and squares

An equilateral triangle (*opposite top left*), or two nested squares (*opposite top right*) both do the same thing—the circle inside each of these figures is exactly half the size of the surrounding circle. This is a geometrical image of the musical octave, where a string-length or frequency is halved or doubled.

Appropriately, the three-dimensional equivalent of the triangle, the tetrahedron, defines the next fractional proportion, one third, as the ratio of the radius of the innermost sphere to that of the containing sphere (*opposite bottom left*). Two nested cubes, or two nested octahedra, or an octahedron nested in a cube (*opposite bottom right*), or a hexagram in a hexagram all produce one third too. The geometric third is musically equal to an octave plus a fifth in harmonic notation. Thus two dimensions quickly define a half, and three dimensions a third.

A close but not perfect marriage occurs between five and eight (*below*), whose geometries often play with one another.

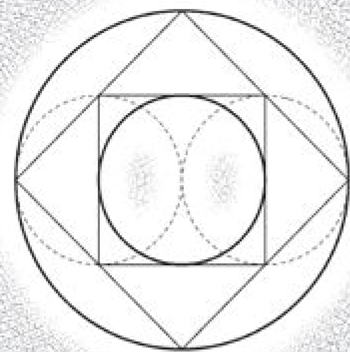




ad triangulum

an equilateral triangle defines two

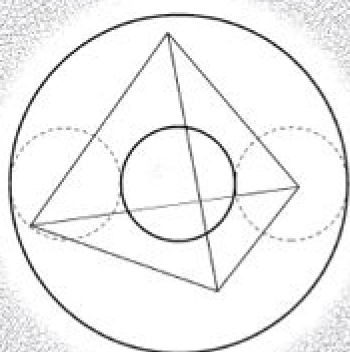
circles, one half the size of the other



ad quadratum

two nested squares define two circles,

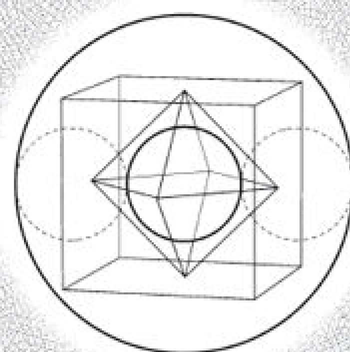
again one half the size of the other



ad tetratum

the sphere inside a tetrahedron is one

third of the size of the sphere outside



ad cuboctum

a cube and octahedron again define

an insphere one third of the outsphere

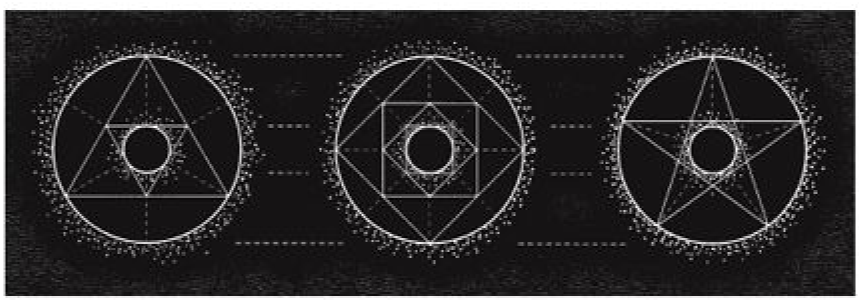
THE SHAPES OF SOUNDS

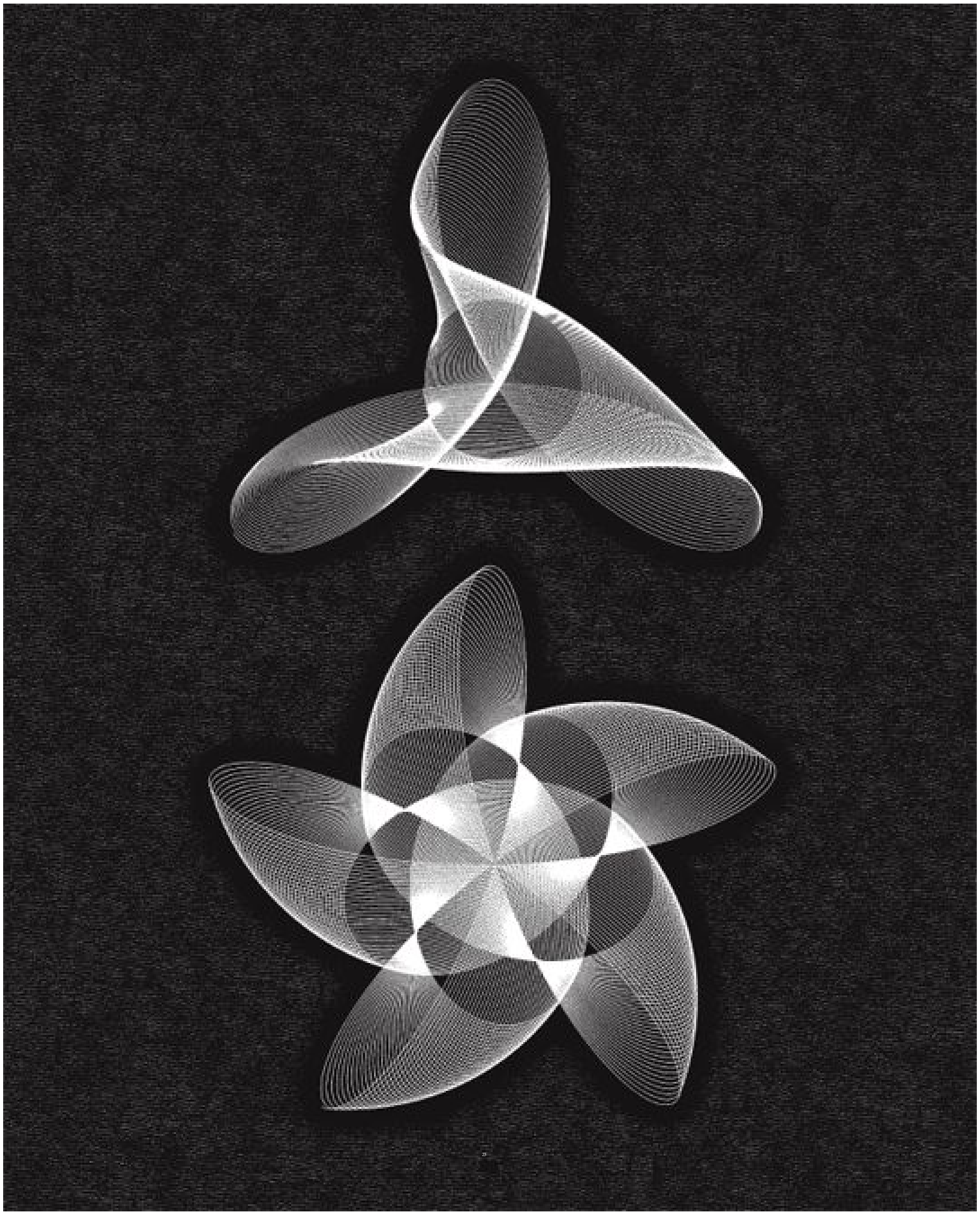
and three quarters

Geometryis‘numberinspace’,musicis‘numberintime’.Thebasicsetofmusicalintervalsistheelementarysetofsimpleratios,1:1(unison),2:1(theoctave),3:2(thefifth),4:3(thefourth)andsoon.The difference between the fourth and the fifth,9:8,isthevalueofonewholetone.Musical*intervals*,likegeometrical*proportions*,alwaysinvolvetwoelementsinacertainratio:twostring-lengths,twoperiods(lengthsoftime)ortwofrequencies(beatsperlengthoftime).Simpleratiossound andlookbeautiful.

Wecanseeharmonicmusicalintervalsasgeometricalshapesbyswingingapeninacircleatonespeed,andatableinancircleatanotherspeed(a*harmonograph*).Shownoppositearetwopatternsfromnear-perfectintervalswherethepenand tableswingcirclesinoppositedirections.Theoctave(*upper*)fascinatinglydrawsasatriangularshape,thefifth(*lower*)asapentagonalform.

Twooctaves,4:1,oraquarter,canbeexactlydefinedbytwotriangles,foursquares,orbyapentagoninapentagram(*below*).





The Golden Section

and other important roots

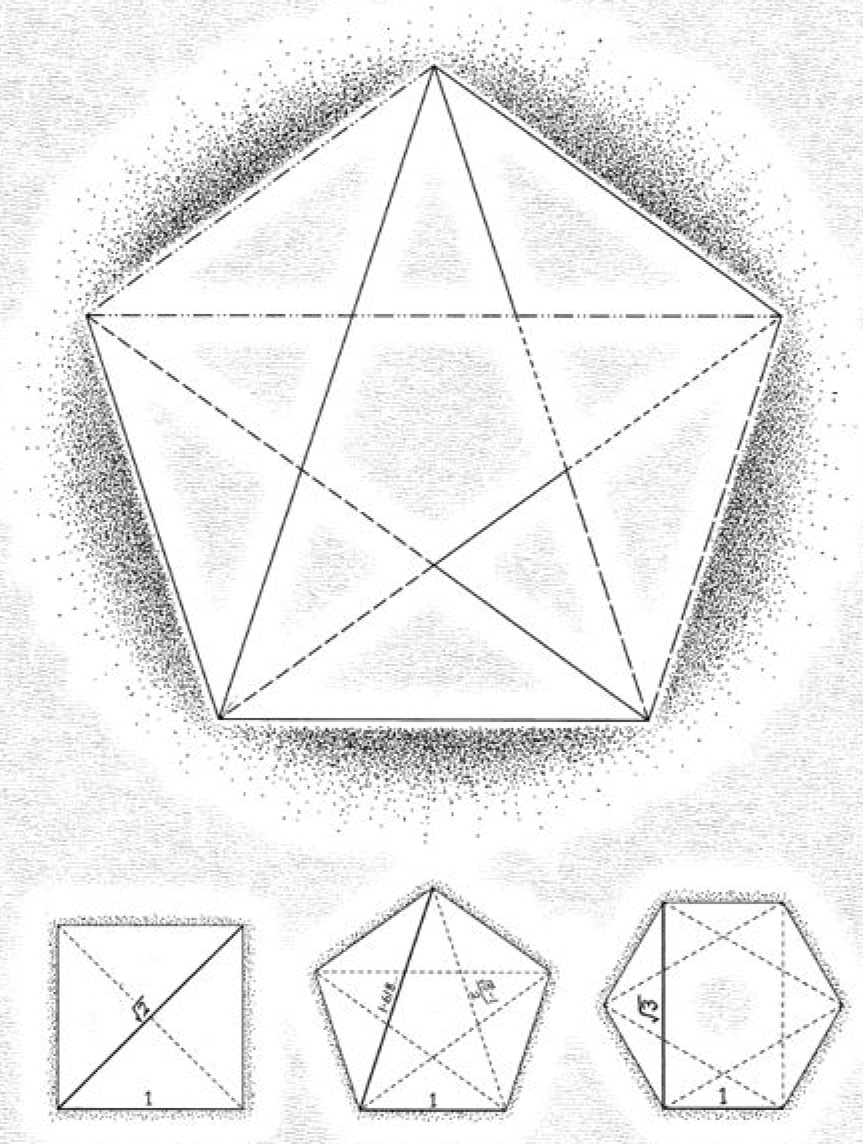
A pentagram inside a pentagon is shown opposite. A simple knot, carefully tied in a ribbon or strip of paper and pulled tight and flattened out makes a perfect pentagon. Try it some time!

In the main diagram opposite you can see that pairs of lines are each dashed in different ways. The length of each such pair of lines is in the *golden section* ratio, 1 : φ, where φ (pronounced *'phi'*) can be either 0.618 or 1.618 (more exactly .61803399...).

Importantly, φ divides a line so that the ratio of the lesser part to the greater part is the same as the ratio of the greater part to the whole. No other proportion behaves so elegantly around unity. For instance, 1.618 x 0.618 = 1 and 1.618 x 1.618 = 2.618. So one over φ is φ minus 1 and φ x φ is one plus φ!

The golden section is one of three simple proportions found in the early polygons (*opposite below*). With edge-lengths 1, a square produces an internal √2, a pentagram 1.618, and a hexagon √3. Although √2 and √3 are found widely in the animal, vegetable and mineral kingdoms, φ appears predominantly in organic life and only rarely in the mineral world. All these proportions are employed in good design. Many familiar objects from cassettes to credit cards and Georgian front doors are φ rectangles.

Neighbouring terms in the Fibonacci Series: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144... (*adding last two numbers to get the next*) increasingly approximate φ. For the keen φ = $\frac{1}{2}(\sqrt{5}-1)$.


√2
1
1.618
1
√3
1

Some Special Spirals

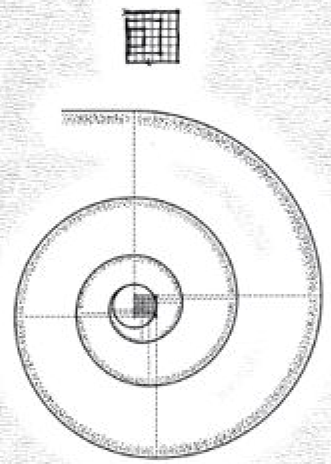
and how to draw them

Spirals are marvellous forms which nature uses at every scale. Three have been selected for this book which give the impression of a spiral from multiple arcs of circles.

The first is the Greek Ionic volute shown top left. This is quite hard to draw and the secret lies in the small 'key' shown above it. The dotted lines in the main drawing show the radii of the arcs and give clues to the centres. It's not as hard as it looks!

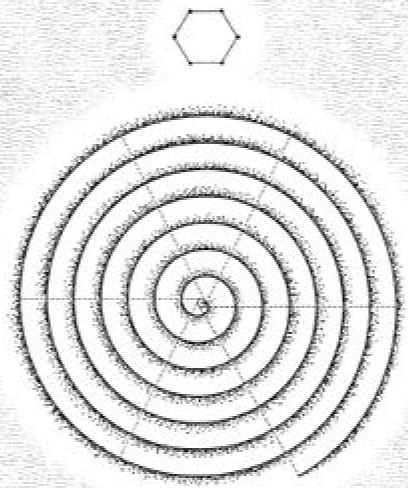
Regular spirals such as the ones shown top right also need a 'key'. This can simply be two dots, a triangle, a square, a pentagon or a hexagon (*as shown*. The more points you have the more perfect the spiral will be. Simply draw the tiny key and open the compasses wide to draw your first arc until it lines up with the next point of the 'key'. Now move the compass point nearer to the arc, reduce the angle to fit and draw the next section. It sounds much harder than it is - if you try it you will soon get the idea. The bigger the 'key' the wider the coils. Now look at the Ionic volute 'key' again - can you see what is happening?

The bottom picture shows a golden section spiral, common throughout the natural world and an example of an exponential spiral. A golden section rectangle has the special property that removing a square from it produces another golden section rectangle, and the golden section spiral is formed by removing successive squares and filling each with a quarter arc.



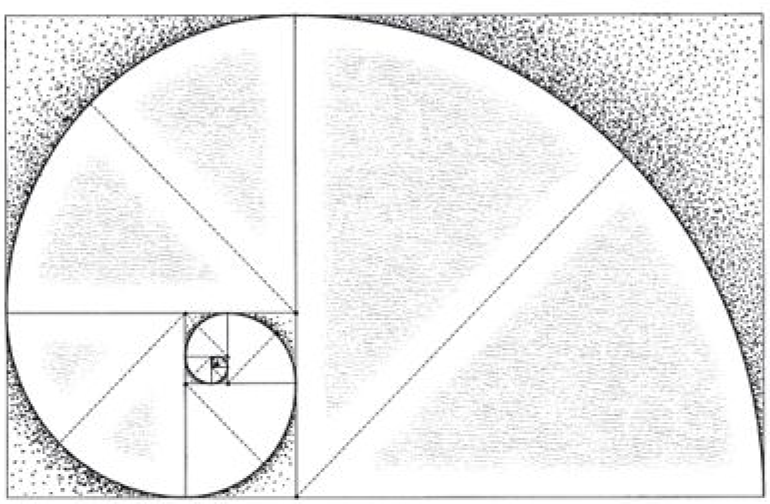
Greek Ionic Volute Spiral

with secret key of compass positions above



Regular Spiral

based on a hexagon of compass positions



Golden Section Spiral

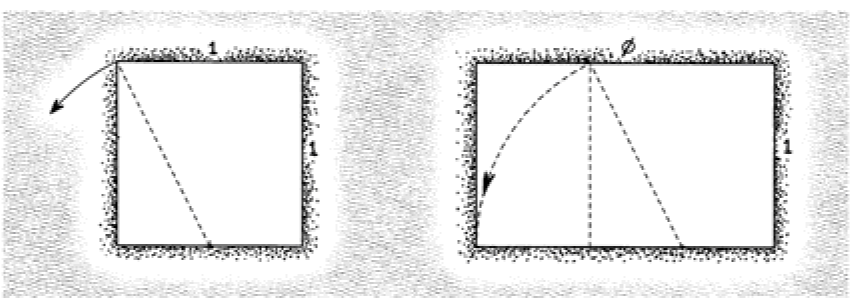
How to Draw a Pentagon

and a golden section rectangle

The method of construction of a pentagram shown opposite is perfect and is from the Almagest of Ptolemy [d. *ca.* 168 AD].

Draw a horizontal line with a circle on it. Keeping the compass opening fixed, place the point at ‹1› and draw the vesica through the centre of the circle. Now open the compasses wide and draw arcs from ‹1› and ‹2› to cross above and below the circle. Use a straightedge to draw the vertical through the centre of the circle. Next draw the vertical through the vesica to produce ‹3›. With the point of the compasses at ‹3› swing an arc down from ‹4› at the top of the circle to give ‹5›. With the point at ‹4› swing through ‹5› to give two points of the pentagon. With the point of the compasses on these new points in turn, swing from the top to find the last two points of the pentagon.

A golden section rectangle, widely used in architecture, is constructed from the mid-point of the side of a square (*below*).



an ancient method for the construction
of a perfect PENTAGON - numbered points
represent compass positions

A Decagon (10 sides) may be easily
constructed from a pentagon - ten
pentagons precisely fit around a
decagon

the pentagon and its starry twin the
pentagram are both symbols of water,
each molecule of which is the corner of a
pentagon, and of the life-force itself

the number five represents the sacred
marriage of two (female) and three
(male) and so symbolises reproduction
and the magical healing arts

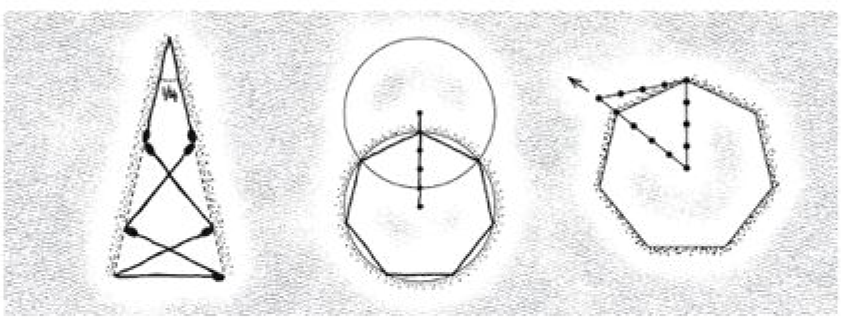
Constructing a Heptagon

seven out of three

Divide a circle into six and draw the primary equilateral triangle. Find the midpoints ‹1› and ‹2› of the triangle's two upper arms and drop two lines down to give two points ‹3› and ‹4› on the base of the triangle, and two on the bottom of the circle. Finally, from the top, swing through the four points on the triangle to give the last four points of the seven on the circle.

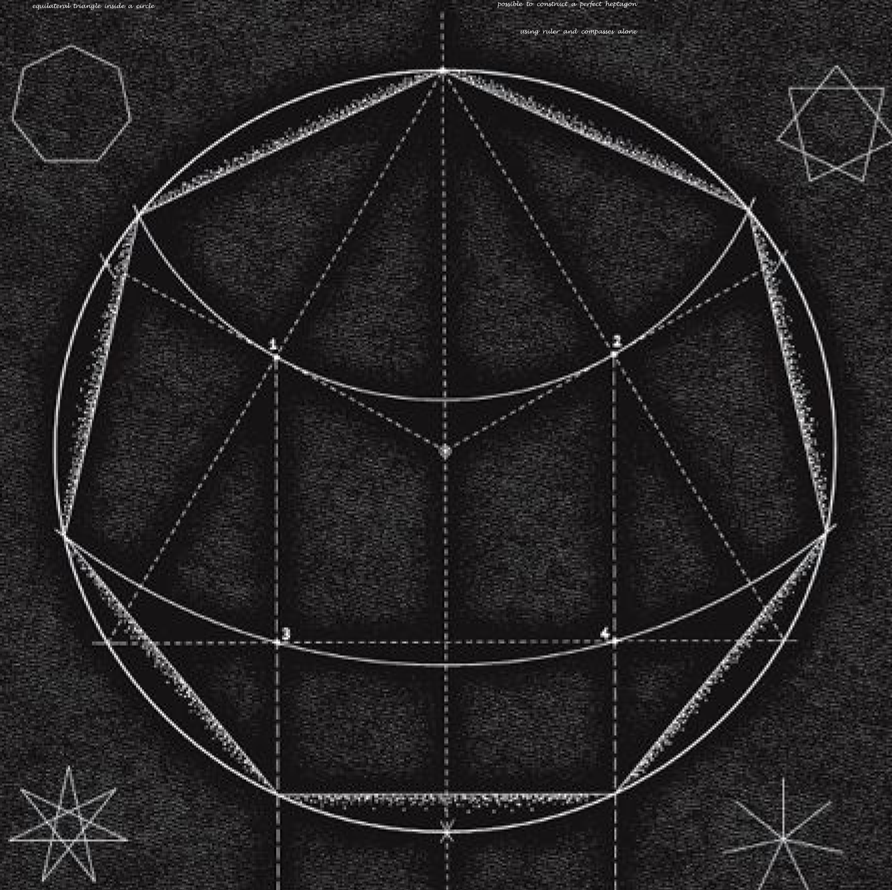
Although it is impossible to draw a precise heptagon using ruler and compasses alone, you can do it perfectly using seven equal rods or matchsticks (*shown below left*). This wedge is an exact fourteenth of a circle, so you need two of them for a one seventh division. More ancient rough solutions use a cord with either six knots or in a loop with thirteen (*below centre and right*).

The ancient builders were amazing surveyors. Avebury stone circles in England are positioned at latitude 51.4°, one seventh of a circle up from the equator and Luxor in Egypt is at a latitude exactly halfway between Avebury and the equator.



a secret method for the construction of a near-perfect HEPTAGON starting with an equilateral triangle inside a circle

introduces no compound errors as each vertex is defined independently - it is not possible to construct a perfect heptagon using ruler and compasses alone



seven symbolises the virgin, having little to do with the other low numbers, and being complete in herself - many traditions revere seven as especially holy

the seven notes of the scale, seven days of the week, seven heavenly bodies of antiquity and seven bodily chakras all indicate the ancient sanctity of seven

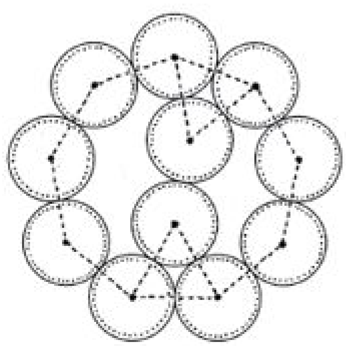
EXTRACTING AN ENNEAGON

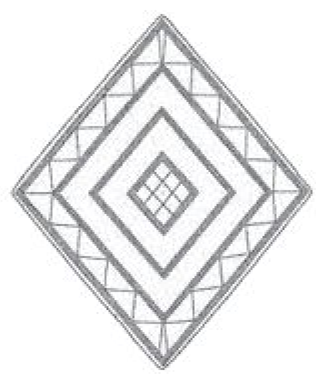
nines and magic squares

The construction shown opposite divides a circle into a near-perfect nine from an initial six-pointed star using three centres.

The digits of many special numbers add up to nine: e.g. 2,160 or 7,920, the diameters of the Moon and the Earth in miles; or 360 and 666 and all pentagonal angles like 36, 72 and 108. All multiples of nine add up to nine. Nine is three times three, or three 'squared'. Many tribal cultures speak of nine worlds, or nine dimensions. Nine coins arranged in a perfect nine-sided figure can contain two more which exactly touch (*below left*).

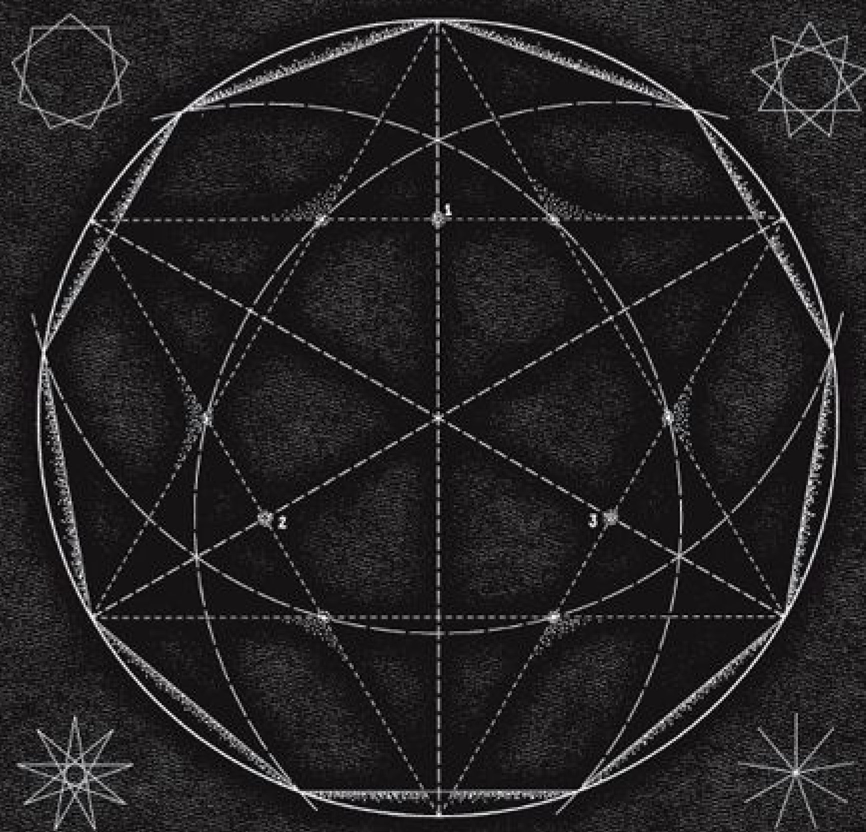
The 5000 year-old golden Bush Barrow lozenge found near Stonehenge (*below right*) has internal angles of 80° and 100°, suggesting nine-fold geometry. Sunrises and sunsets at the latitude of Stonehenge vary over 80°, and moonrises and moonsets over 100°, so this was probably quite a useful object.





John Michell's method for the construction of a near-perfect ENNEAGON starting with a hexagram inside a circle

a simple and memorable method, accurate enough for most practical purposes



the symbolism of nine is laden with initiatory veils with stories of nine worlds or realms in many shamanic traditions

nine is the square of three and along with eight, the cube of two, is very important in Eastern cosmologies

RABATMENT

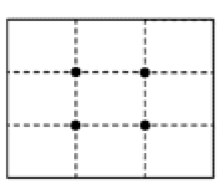
and the rule of three

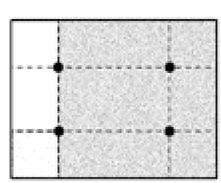
Painters often use 'the rule of three' to make their pictures sing. They divide their canvas into three, horizontally and vertically, making nine small versions of the original rectangle (*below left*). The four intersections are excellent places to focus items in the design and are widely used by artists. By contrast, items placed on centre lines seem contrived in the composition, too obvious.

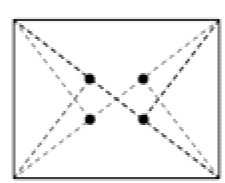
Another trick, *rabatment*, draws a square within the rectangle, and uses the lines produced as focal axes (*below centre*). In Golden rectangles the space left over from a square is another Golden rectangle so the process may be continued indefinitely.

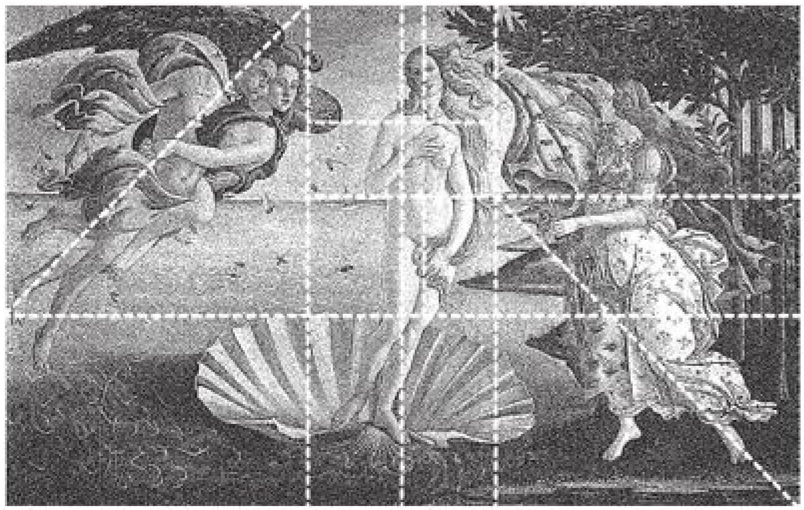
Occult centres (*below right*) are found by using the diagonals of a rectangle and right-angled triangles from the other corners.

Dividing a line into 2 or 3 parts uses the first few steps of the Fibonacci sequence, 1, 2, 3, 5, 8, 13, 21, and so on, where adjacent terms home in on the Golden Section 0.618. Some painters use $\frac{1}{5}$ divisions or even the Golden Section itself. Opposite we see Botticelli using a Golden rectangle reduced by rabatment in stages to compose his painting, while Grimshaw uses a rectangle between f and √2, with halves and Golden lines as guides.

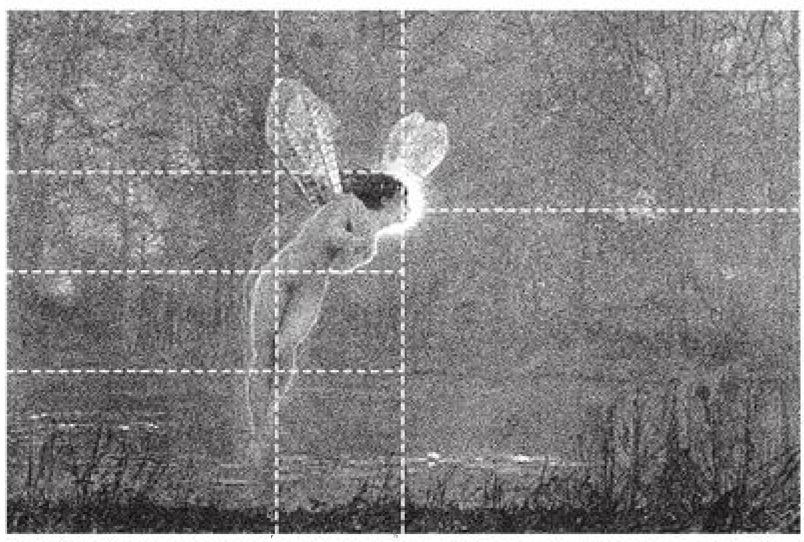








Above: Botticelli's Birth of Venus. A Golden Section rectangle with interior squares define the horizon, spine, navel and other features. Below: John Atkinson Grimshaw's Iris fairy is positioned using centre lines and golden divisions. The fairy exists in half the painting.



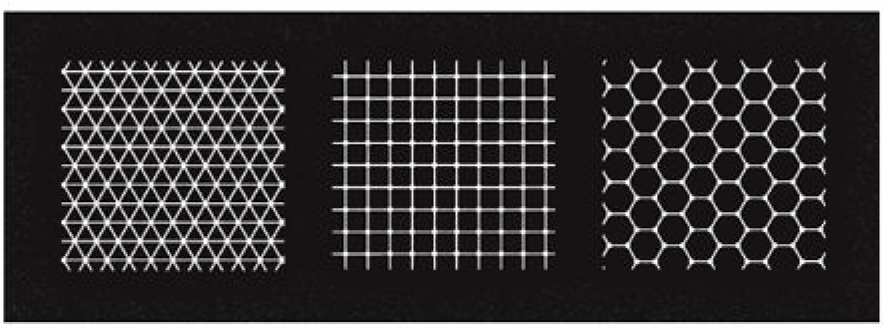
Simple Tilings

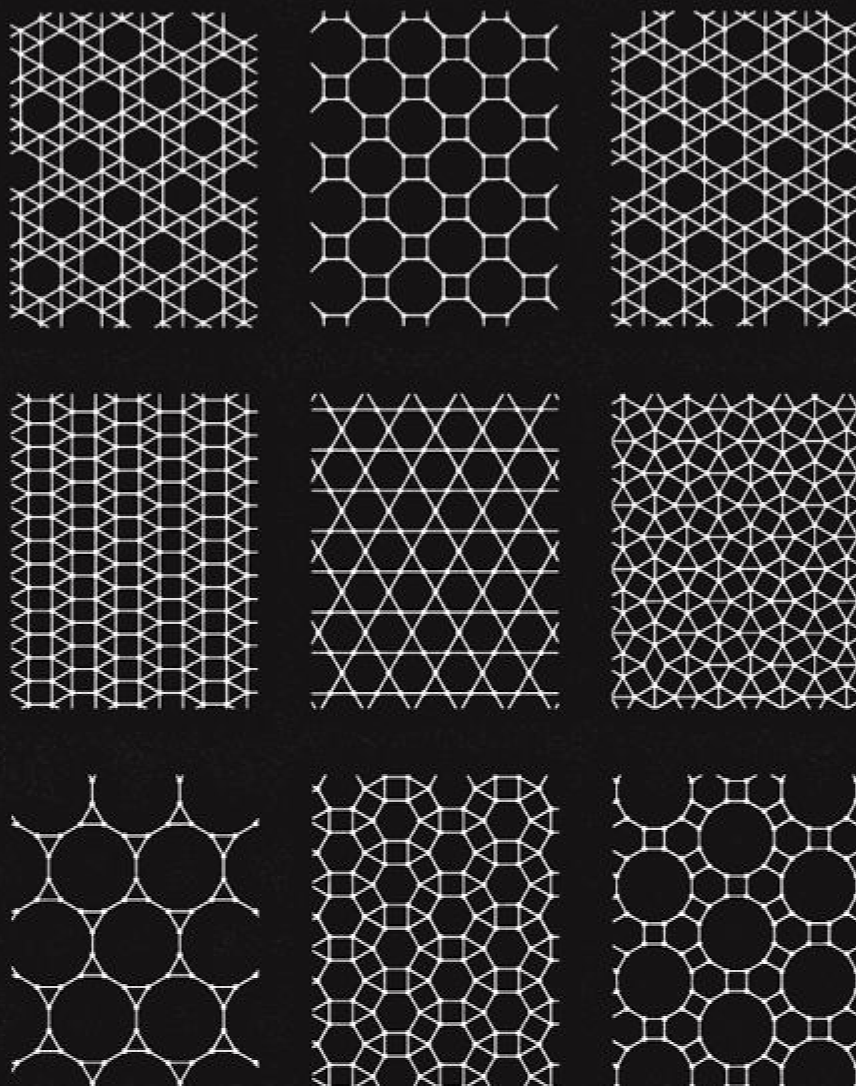
repeating patterns over an infinite surface

A regular tiling (or tessellation) of the plane occurs when the same regular polygon is used to fill the plane, leaving no spaces. Only three of these are possible (*shown below*). A semi-regular tiling allows for more than one type of polygon but insists that each vertex is the same. For example, in the central pattern opposite every vertex is a meeting of two hexagons and two triangles. Eight semi-regular tilings are possible and all are shown opposite (though the top left and top right grids opposite are left- and right-handed versions of each other and count as one).

Some designs can be filled in further - as we saw on page 11, dodecagons are just made of hexagons, triangles and squares, and hexagons can be reduced to triangles. Triangles and squares can go on to do the most amazing things together (*next page*).

Octagons only tile with squares (*opposite top centre*). Pentagons do not fit together easily on the plane, preferring the third dimension (*see pages 12–13*). Heptagons and enneagons stand aloof.





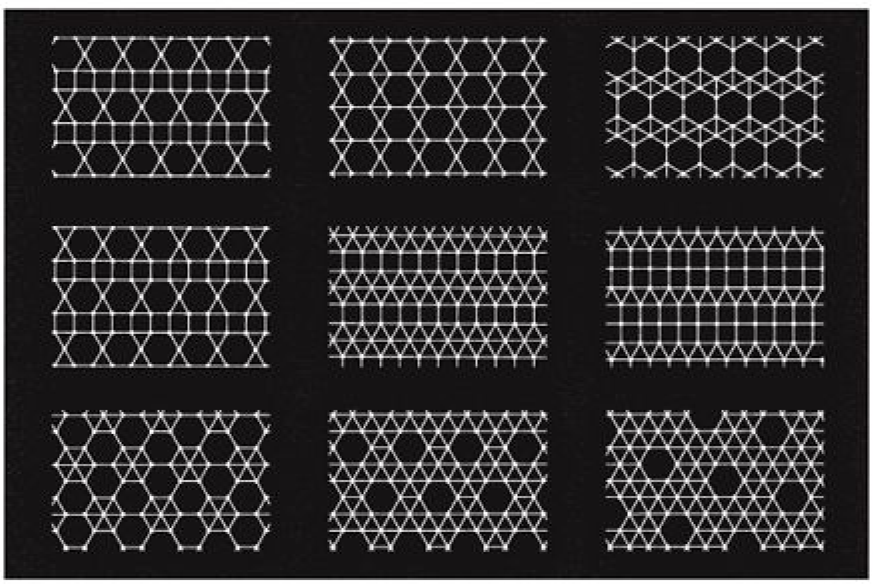
Further Tilings

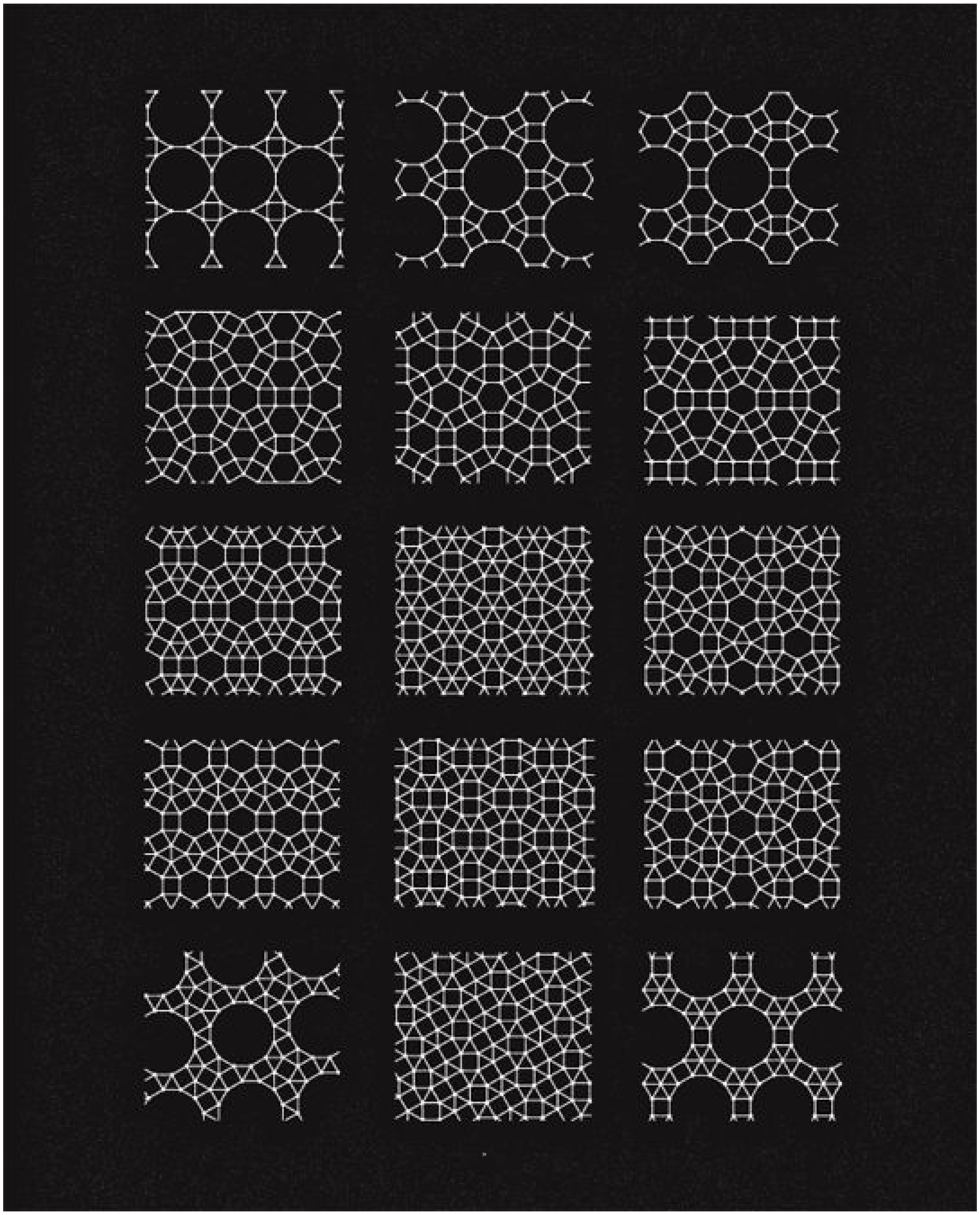
more fun in the bathroom

There are twenty demi-regular tilings (where two vertex situations are permitted) and most of these and a few other interesting ways to tile the plane are shown on these two pages.

These tessellations form the basis for pattern construction in many traditions of sacred and decorative art. They can be found underlying Celtic and Islamic patterns and in the natural world they appear as crystal and cellular structures. William Morris used them widely for his repeat wallpaper and fabric designs. Their uses are limited only by your imagination!

On the next page we see one of these grids put to use.





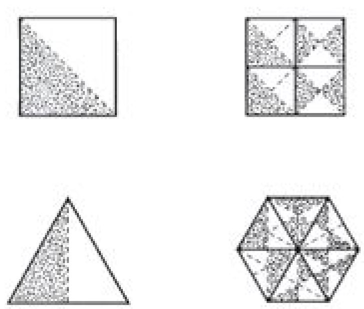
THE SMALLEST PART

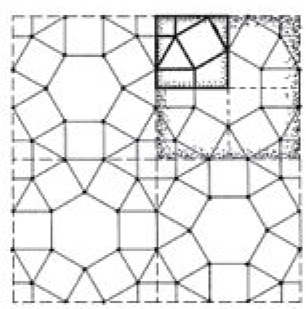
reversible stencils and rotable blocks

Many of the semi- and demi-regular grids can be reduced to a simple square or triangular unit which can then be reflected or rotated to recreate the whole pattern. Often these repeat triangles or squares are quite small. However, in practical applications it is often easier to rotate a printing block or stencil than it is to reflect it - and in such cases one has to take care to draw a slightly larger stencil or carve a slightly larger block.

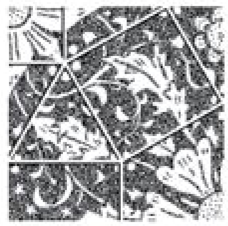
The design shown opposite is based on one of the grids on the previous page. It is produced by rotation *and* reflection of the primary unit (*opposite top and below right*).

Squares and equilateral triangles can themselves often be halved to produce smaller triangular units (*below left*). But, again, take care and think about what you are doing; for instance you cannot do this with the example shown opposite - can you see why?

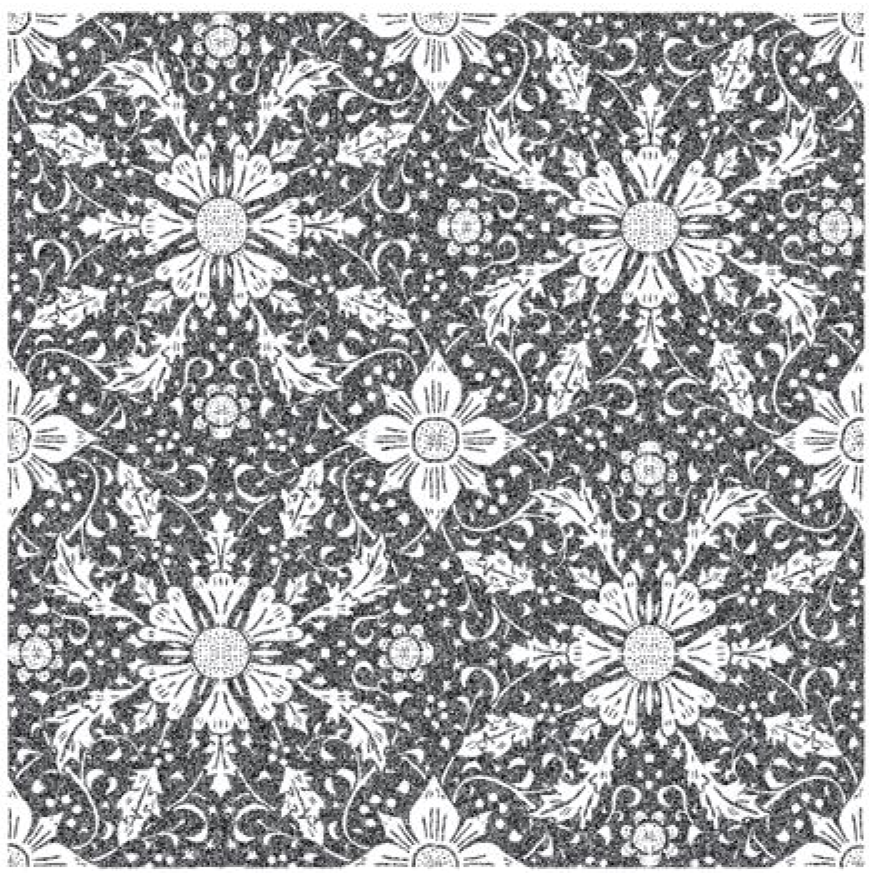




The smallest unit of the pattern below, drawn at the same scale, and howing the grid lines which have informed its design.



Notice how leaves and petals have been positioned along the edges of the tile, with the centres of flowers at the centres of rotation points.



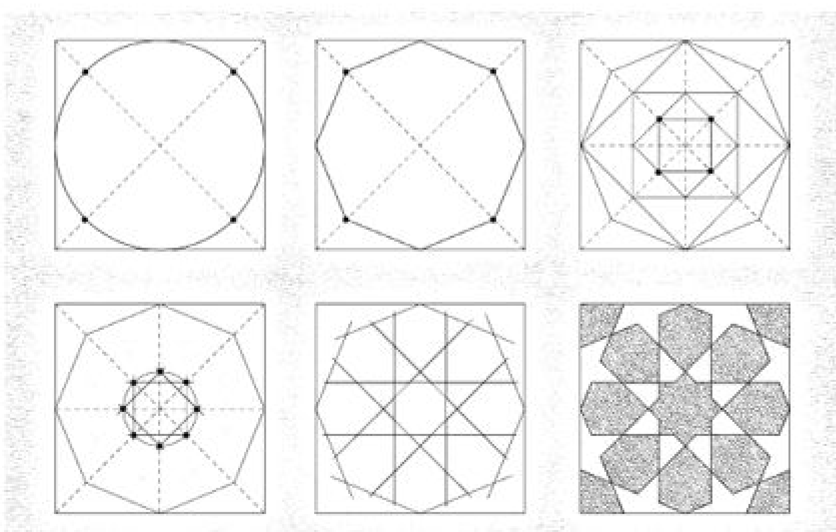
Islamic Designs

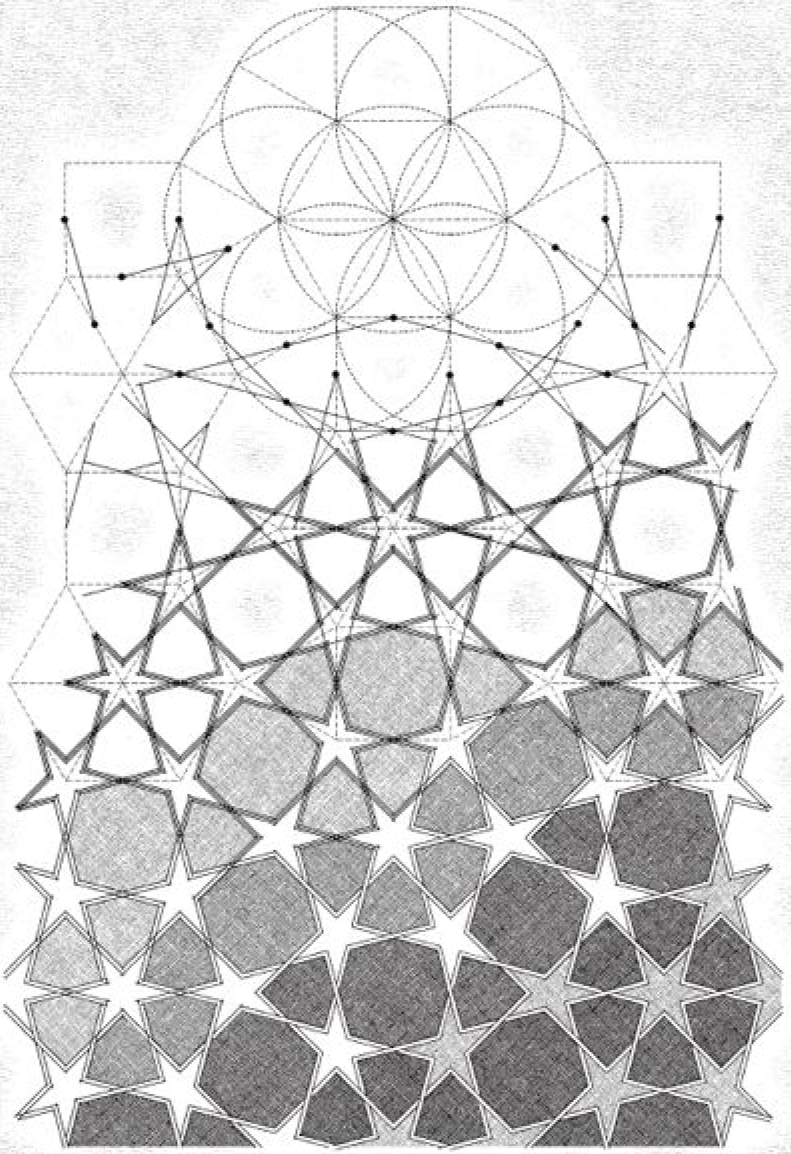
stars are born from subgrids

Islamic patterns speak of infinity and the omnipresent centre.

For the pattern opposite, start with six circles round one, developing a grid of overlapping dodecagons from triangles, squares and hexagons (*see pages 11 and 37*). The key points now are halfway along the side of every polygon. These are joined up in a special way and extended as shown. Below an octagonal design is extracted from a square tile. Many beautiful patterns are sitting in every simple subgrid, just waiting to be pulled out.

The subgrids themselves are rarely shown in traditional art. They are considered part of the underlying structure of reality, with the cosmos ('cosmos' means 'adornment') overlaid.



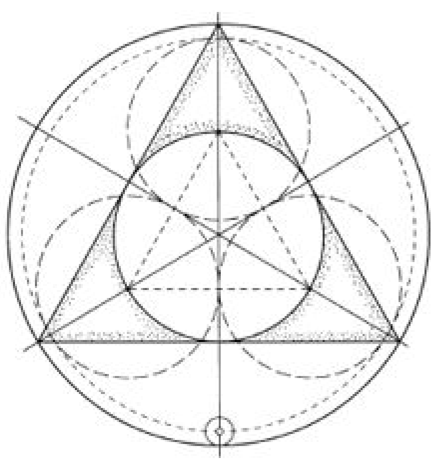


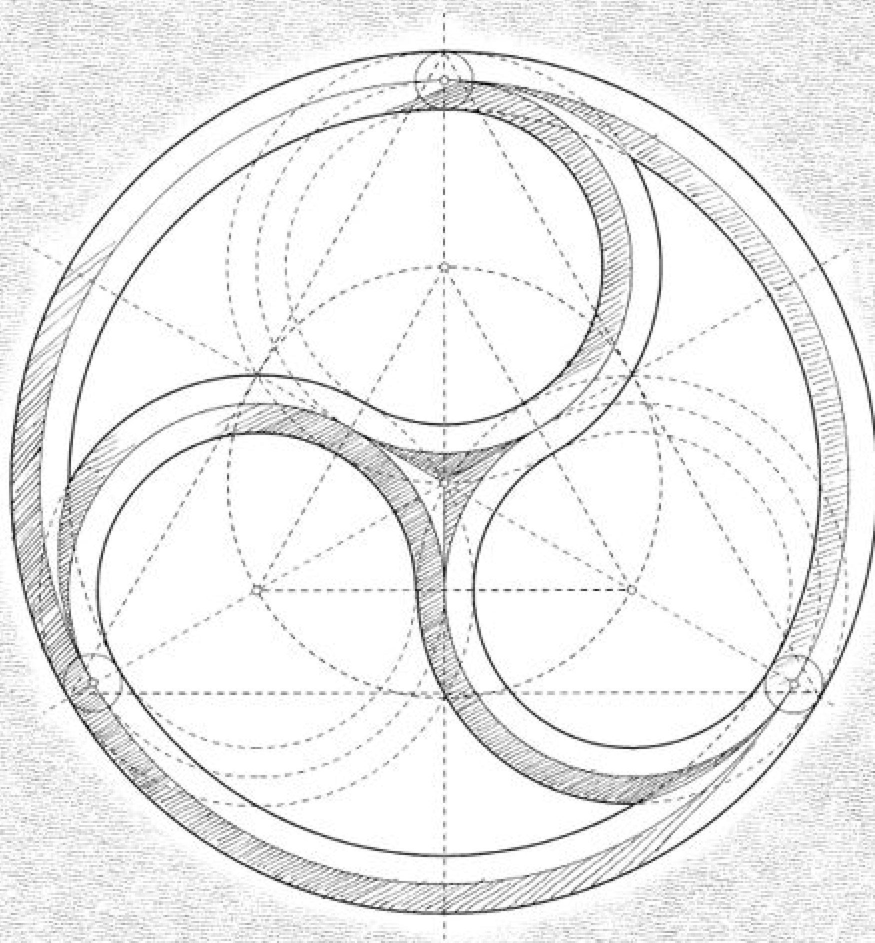
A Church Window

not far from the Isle of Man

A typical piece of church window masonry is shown opposite. It is a very beautiful design and the sort of thing that can be seen in many churches. There are a number of ways of drawing it but see if you can follow its construction from the diagram below, which begins with the enclosing circle. Notice how every detail is defined by the geometry.

Draw a large circle and divide it into six. In this circle draw a large triangle and fit a circle inside it to give the centres of three touching circles (the centrelines of the tracery). Notice how these do not quite touch the outer circle. A small circle (*at the bottom of the image below*) then assists in giving the width of the stone tracery itself. The design speaks of the implicit trinity in unity.





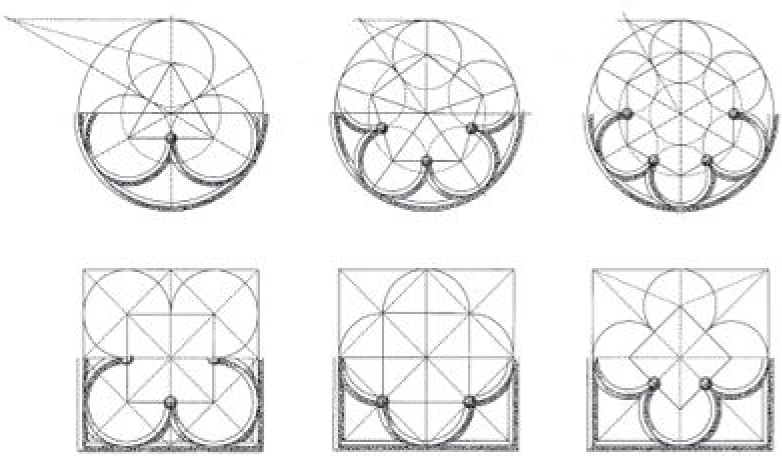
Trefoils and Quatrefoils

the art of framing the light

Everything is made of light, all matter is, and without matter there would be no sound. Atoms and planets arrange themselves in geometrical patterns. How profound then is a window, which allows the passage of light into an otherwise dark space.

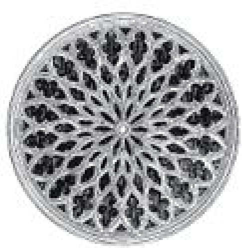
The designs of church windows follow many rules, forms and traditions, and some clues are given on these pages. The easiest to draw are the three quatrefoils (*bottom row of this page*).

The south window of Lincoln cathedral with its striking double vesica is shown opposite, and below it three famous and very early west windows, from Chartres, Evreux and Rheims cathedrals. A good balance is kept between line and curve.



Lincoln

Chartres



Evreux

Rheims

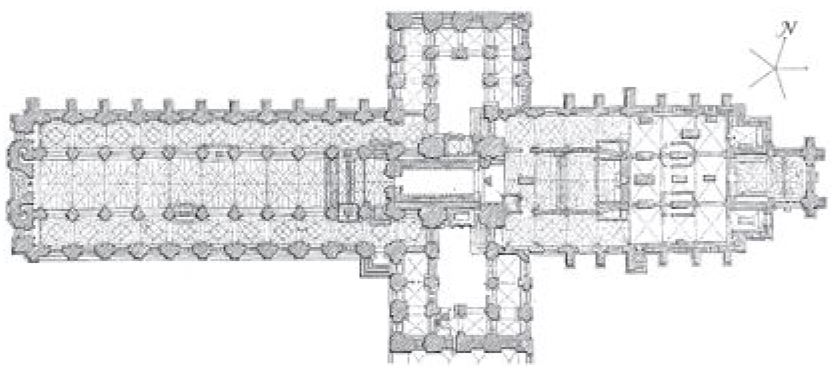
Stone Circles & Churches

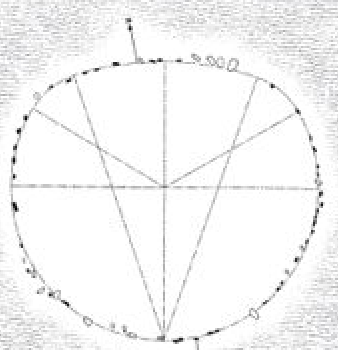
vesicas in action for over 4000 years

Four flattened stone circles are shown opposite with their consistent geometry as discovered by Professor Thom. On the left are examples of the type-A shape, on the right type-Bs. The vesica-based constructions are also shown (*see page 6 for vesica*).

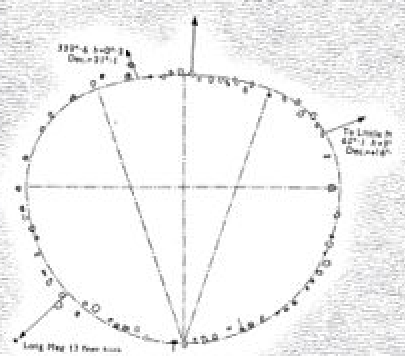
Shown on this page is the ground-plan of Winchester Cathedral. An interplay of simple *vesica*-based triangular and square systems, *ad triangulum* and *ad quadratum*, underlies the plans of many ecclesiastical buildings (*see top row page 21*).

The design of a sacred building, whether church, stone circle or temple, requires the designer to marry the universal symbolism of the geometrical moves he or she is making with the specific religious language. Local factors also go into the cauldron, e.g. Sun, star or Moon rising and setting positions, or nearby sacred hills, springs or leys. Below we see Winchester Cathedral's axis pointing 72° from north, creating a magical magnetic pentagram.

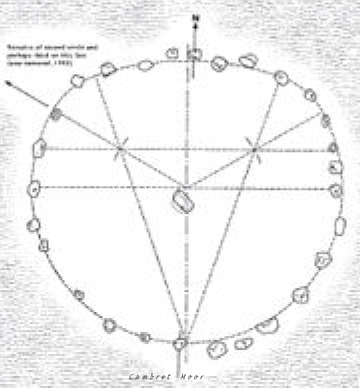




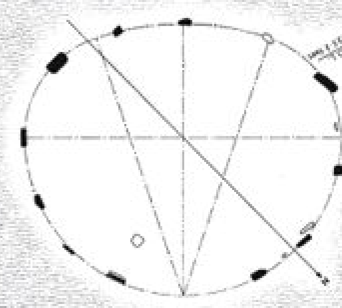
Dinnever Hill



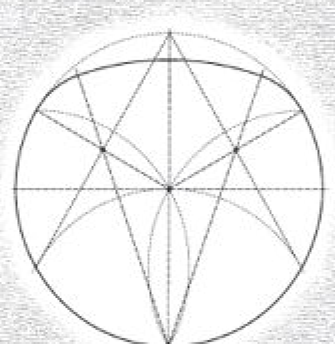
Long Meg



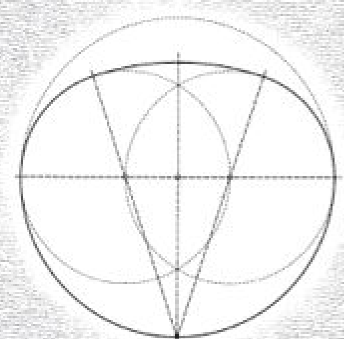
Cambret Moor



Barbrook



Type-A flattened stone circle



Type-B flattened stone circle

Delightful Arches

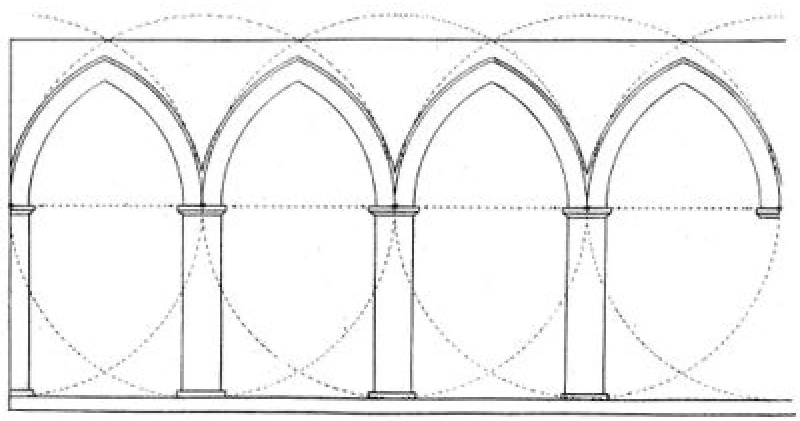
some simple ways through

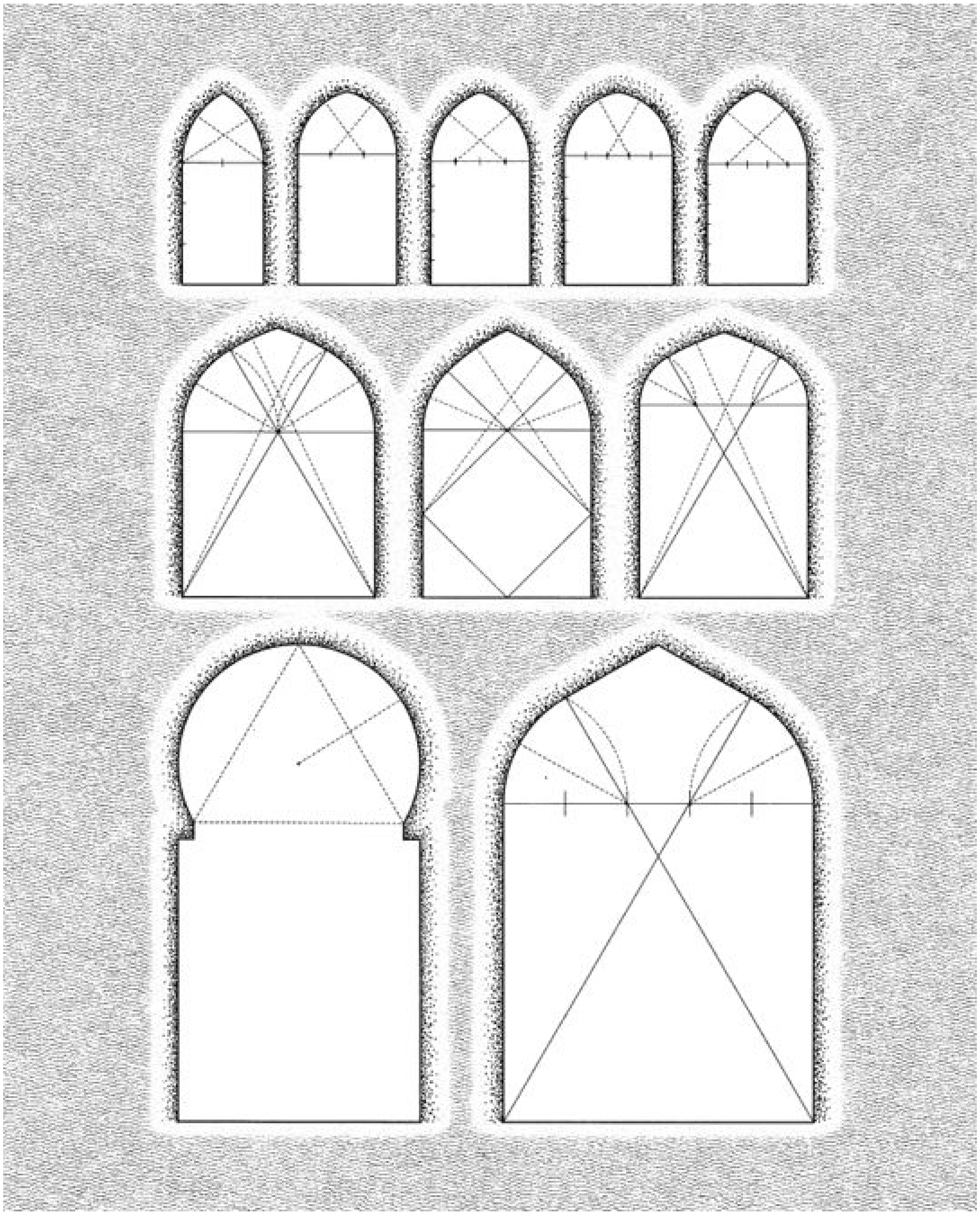
Arches take remarkably similar forms all over the world and a few are shown here. Living trees often make the best arches.

The top row opposite shows five two-centred arches. Their span has been divided into 2, 3, 4, 5 and 5 again. The straight dotted lines show the radii of their arcs. The heights of arches can vary but for these five their heights are defined by a rectangle which gives a musical interval, thus 2:3, 3:4 and so on (*page 22*).

The second row of arches opposite are four-centred. The curve of the arch changes at a position given by the undashed line. Ideas for defining their heights are also given.

The bottom two arches opposite are a horseshoe arch (which can also be pointed) and a pointed arch. The pointed arch seems to turn up - 'the return' - but the lines are actually dead straight.





A Celtic Spiral

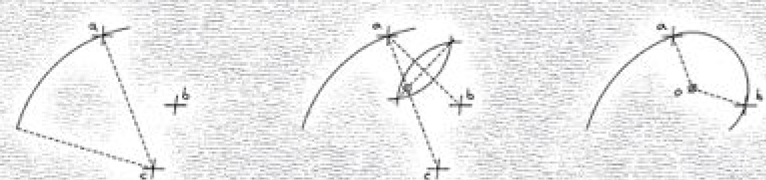
and how to tighten up your curves

The design shown opposite comes from a 2,000-year-old four inch bronze disc found on Loughan Island, Northern Ireland. It is an exceptionally beautiful example of the early Celtic style. As we have already seen with stone circles and arches, the seamless conjunction of multiple arcs can be highly aesthetic and it reached its perfection in the early Celtic period.

Many early Celtic pieces show evidence of compass use and the final drawing for this disc required no less than 42 separate compass positions! The master artists who created these designs started with a basic geometric template, such as a touching circles pattern, then sketched their forms before returning to geometry to tighten everything up so that their curves all became arcs, sections of circles. This gives a tautness to the curves. In this way intuition and intellect work together.

The lower sequence of pictures shows how to plot arcs through points. The first diagram shows an arc centred on ‹*c*›. We want the arc to change effortlessly at ‹*a*› and then pass through ‹*b*›. What do we do? Find the perpendicular bisector between points ‹*a*› and ‹*b*› by opening the compass, describing two equal arcs from ‹*a*› and ‹*b*›, and drawing the line through their intersections (*centre*). This cuts the ‹*a-c*› line at a new point ‹o› which then becomes the centre we were looking for (*right*).

All of the beautiful curves in the Loughan Island disc are drawn and tautened in this simple and elegant manner.



The idea that circles or arcs of circles are the only truly heavenly curves finds its way into many philosophies of sacred art. The technique above may be used in any application to refine a curvy sketch or design and give it that elusive magical quality.

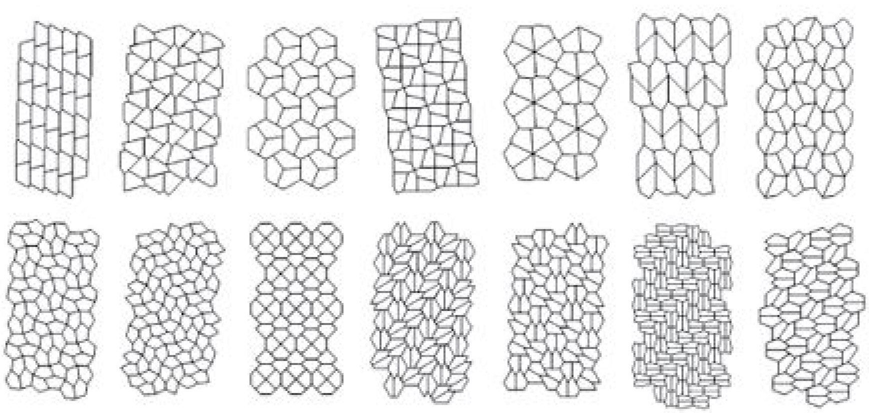
Pentagonal Possibilities

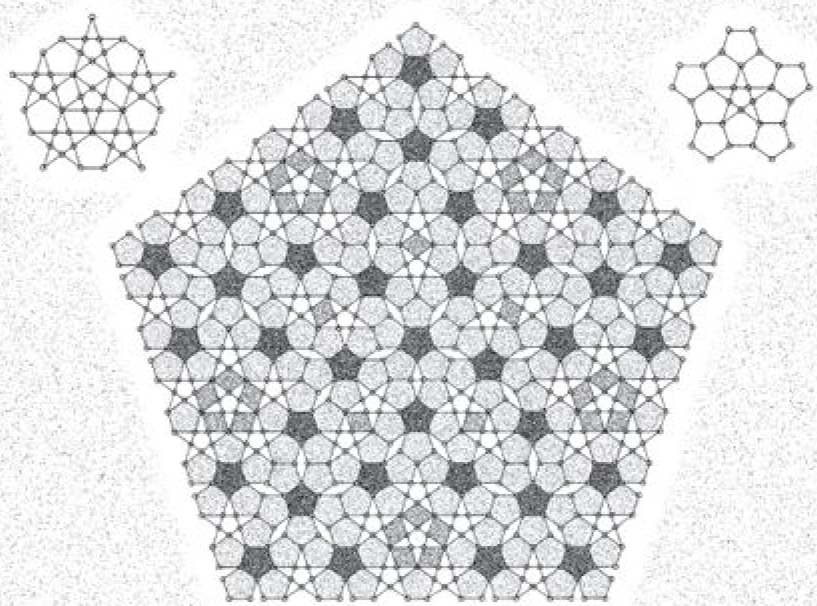
those phantastic phizzy phives

Although the regular pentagon does not tile on the plane, it does do various other interesting things. One of these, taken from studies by Johannes Kepler [1571–1630], involves a 'seed' pattern which grows from the centre (*opposite top*). The grey pentagons leave spaces which are bits of pentagrams and vice versa. The design is riddled with examples of the Golden Section. Two other seeds are shown.

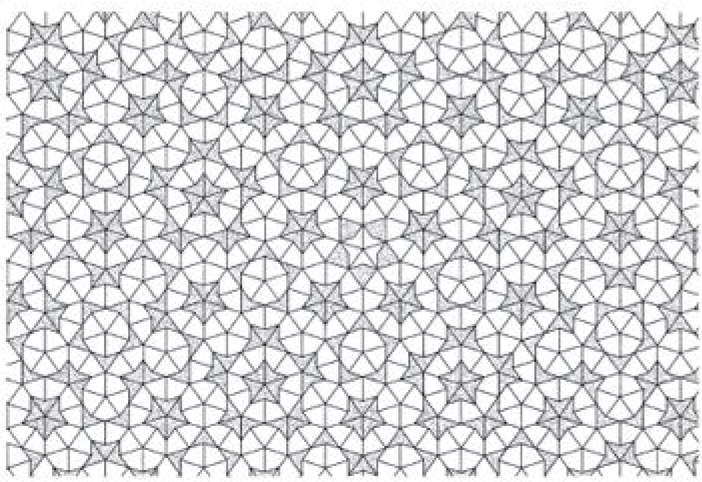
In Roger Penrose's famous pentagonal tiling (*lower, opposite*) two shapes tile to fill the plane with aperiodic 'not-repeating' pentagonal elements at all scales. These patterns have recently been found to underlie the nature of most liquids. They are, for instance, cross-sections through water.

Shown below are the fourteen types of irregular convex pentagon which tile the plane.





Kepler's early 17th century infinite pentagonal system and below, Penrose's 20th century pair of tiles which fill the plane



THE SEVENTEEN SYMMETRIES

from slide, spin and mirror

The Arab alchemist Jabir ibn Hayyan, known in the West as Geber, regarded 17 as the numerical basis of the physical world.

Using a very simple sample design the next three pages explore the three basic operations of rotation, reflection and sliding. These, combined with the three regular tilings, give seventeen patterns which are shown below, opposite and on the next page.

This visual key can be very useful when creating repeats for fabric or pottery patterns (*see too pages 40–41*). 'Pattern', by the way, comes from the latin word pater, meaning 'father', in the same way that 'matrix' comes from mater, meaning 'mother'.

Remember, not all stencils can be turned over (reflected) without making a mess so choose your repeat units with care.

And on that rather practical note this dense little book on one of the oldest subjects on Earth has now reached its end; I hope you have gleaned enough ideas from it to create something good, true and beautiful next time you get designing!